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PHYSICS-INFORMED NEURAL NETWORKS FOR NARROWBAND SIGNAL PROPAGATION MODELING

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ABSTRACT

Background. Physics-informed neural networks (PINN) demonstrated strong capabilities in solving direct and inverse problems for partial differential equations. In this study, the focus is on applying PINNs for the approximation and extrapolation of narrowband signal propagation. This effort is motivated by the potential to reduce measurement and numerical costs in applications such as acoustic and electromagnetic beacon-based navigation systems. These systems aim to map environments and track object trajectories by leveraging wave propagation data.

Materials and Methods. The propagation of harmonic waves through a medium can be described using either the wave equation or the Helmholtz equation. To establish a connection between these equations, the Fourier transform is employed. PINNs are trained in the time or frequency domain to predict wave propagation characteristics such as amplitude and phase. The study compares the performance of PINNs against conventional neural networks.

Results and Discussion. The study finds that PINNs exhibit superior performance over conventional neural networks when training data points are separated up to the Nyquist rate. In the time domain, PINNs accurately predict epy phase up to a distance of one cell except for the direction to the source. However, amplitude predictions are less accurate, with errors below 20% up to a distance of 0.5 cells. For larger amplitudes, the model struggles to provide reliable predictions. Training PINNs in the frequency domain requires less computational resources, but performance is worse than in the time domain.

Conclusion. PINNs offer promising advantages for modeling wave propagation in narrowband signals, particularly in scenarios where measurement data is sparse or local. They can increase resolution, reduce the volume of required data, and optimize computational efficiency. Despite their limitation, there is a difference in solutions between the time and frequency domains due to the nonlinear nature of NN. Future work could address the accuracy of predictions through better network architectures or hybrid approaches.

Keywords: Physics-informed neural networks, PINN, waves, super resolution, deep learning, fast Fourier transform

INTRODUCTION

Physics-informed neural networks (PINN) demonstrate good results in solving a particular partial differential equation (PDE), e.g., solving Burger's equation [1]. Unlike traditional neural networks, PINNs embed the governing equations of physical systems into their loss functions, ensuring that the network's predictions are consistent with the underlying physics. Also, this approach is capable to solve inverse problems [2] in different areas. By embedding the governing equations of physical systems (PDEs) into the training process, they achieve better performance in modeling complex systems. This is a promising method that is considered to be applied for quantum computing [3]. However, background investigation of PINNs is not a complicated topic, and this area is rapidly being investigated to understand its limitations.

This paper investigates the capabilities of approximation and extrapolation of narrowband signal propagation using PINN to minimize measurement and numerical costs. Approximation of measured data using PINNs allows for an increase in resolution without extra data sets [4], but it considers convolutional neural networks and conclusions cannot be directly transferred to conventional PINNs that use fully connected layers. On the other hand, such a fully connected PINN can generalize multiple solutions and do generalization [5]. However, such generalization requires multiple measurements that are done in the whole space of interest. This opens the question about extrapolation that is based on data points that do not cover the area of interest. A typical neural network is not good for extrapolation tasks and can produce arbitrary results. These results can vary from multiple factors, e.g., weight initialization, etc. There were investigations of ill-posed problems for near-wall blood flow from sparse data [6] that solve the Navier–Stokes equations. However, extrapolation without specifying boundary conditions is more difficult. Also, there is a large interest in reducing computation resources by training multiple PINNs [7], and this approach may be applied to reduce the number of measurements too.

The motivation for this work stems from practical challenges in navigation systems and wireless communication applications. Some navigation systems require the ability to construct navigation maps based on acoustic and electromagnetic beacons. Also, it can be helpful to do tracking of trajectories of moving objects in wireless applications [8]. These systems require the prediction of signal parameters such as phase and time delay, which are further used to infer distances. Accurate predictions depend on high-resolution data, which is often limited in real-world scenarios due to cost constraints. PINNs offer a potential solution to this problem by increasing resolution through intelligent interpolation and extrapolation without requiring additional measurement points.

MATERIALS AND METHODS

Theoretical Framework

The simplest model to investigate narrowband signals is wave propagation in a medium. It can be described by the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u, \tag{1}$$

where u is the wave function;
c is the wave speed; and
∇^2 is the Laplacian operator.

If the propagation model does not contain nonlinear effects, steady-state conditions can be assumed, and the Helmholtz equation is used:

$$\nabla^2 u + k^2 u = 0, \tag{2}$$

where $k = \omega / c$ is the wave number, and ω is the angular frequency.

The Fourier transform provides a bridge between the time-domain wave equation and the frequency-domain Helmholtz equation. This relation is used to compare trained PINNs in different domains (Fig. 1). Note that the similarity of solutions in the time and frequency domains is not guaranteed because of using a neural network that is a nonlinear approximator.

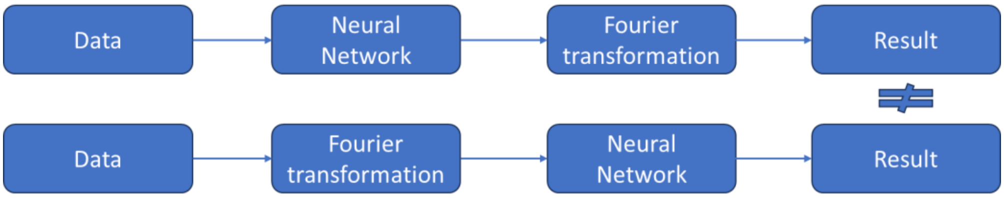

Fig. 1. Comparison of wave propagation in time and frequency domains.

To simplify the investigation, the propagation model of a harmonic wave in a 2D medium is used. The harmonic wave source is located at a fixed point in the domain, and the propagation is computed analytically with a defined spatial and time step. The source is modeled as a harmonic excitation at a fixed point (x_0, y_0) in the domain:

$$u(x,y,t) = \frac{a}{\sqrt{\rho}}\sin(k\rho - \omega t), \tag{3}$$

where $a/\sqrt{\rho}$ is the amplitude of the source wave;
$\rho = \sqrt{(x-x_0)^2 + (y-y_0)^2}$ is the distance from the source of the harmonic wave.
The computed points are used to generate training and test datasets.

The simulation domain is divided into two parts: the core part, which has discretized points from 0 to 1, is used for training and verification of increasing resolution; the area outside of the core is used for verification of extrapolation. The core area has a length of 2.5 wavelength in x and y directions. Data in the core area that is marked by the red rectangle in Fig. 2 has 32 points in the time domain and different discretizations in the space domain, e.g., 32x32 or 8x8. Such greed in the space domain is the worst case for increasing resolution and simple to make conclusions.

The training data are generated for two cases: a single harmonic source that uses the equation above, and two harmonic sources in opposite phase. The goal of the PINN is to predict the wave field across the entire domain, including areas where data are not available.

PINN Architecture and Training

The PINNs used for this study are a fully connected neural network with the following architecture (Fig. 3):

- Input layer: x, y and t for time domain investigation and x, y for frequency domain.
- Hidden layers: three fully connected layers with 64, 128, and 64 neurons, using the Tanh activation function.

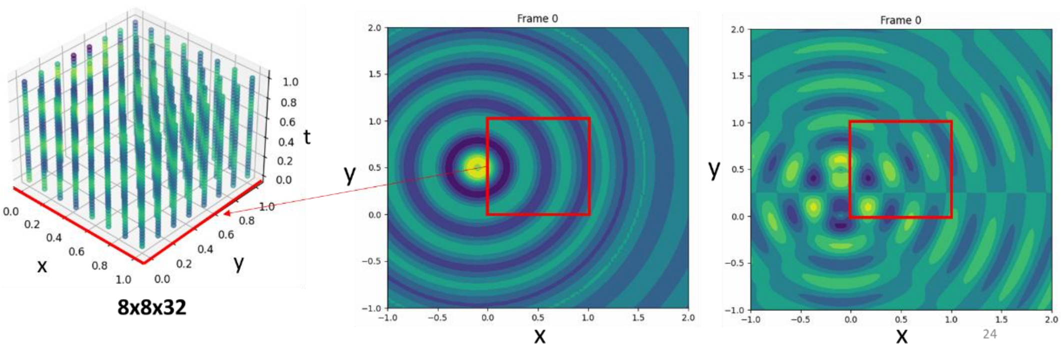

Fig. 2. Input data for neural networks training for single and two sources models.

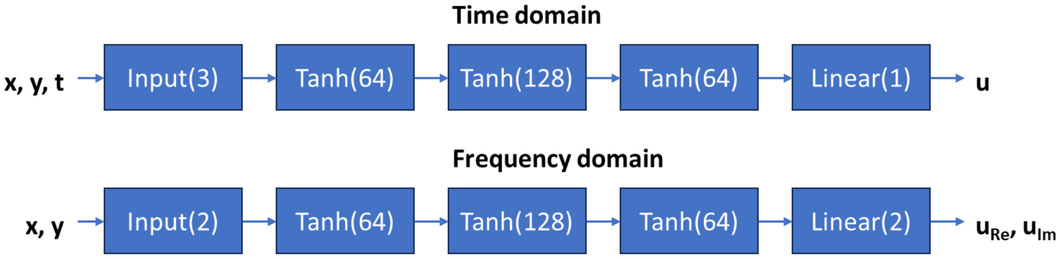

Fig. 3. Architecture of used neural networks for training in time and frequency domains.

- Output layer: one neuron for representing $u(x, y, t)$ in the time domain or two neurons for complex valued $u(x, y)$ in the frequency domain.

The same architecture is used for conventional neural networks with data only losses L_{data}. On the other hand, the PINN loss function used in training combines data loss and physics loss:

$$L = L_{\text{data}} + L_{\text{physics}}, \tag{4}$$

where L_{data} represents the error between the model prediction and the analytical data; L_{physics} represents the error in satisfying the governing PDEs.

The physics loss term is computed by substituting the neural network output into the PDE. Note that boundary conditions and initial conditions are not given separately and are partially present in the analytically computed data.

The ordinary NN and PINN models were developed using PyTorch version 2.5.0 [9].

The training data consists of simulated wave propagation data generated from the wave equation and the Helmholtz equation, which are given above. Physics loss is given in the whole area of interest. Different PINNs are trained in the time or frequency domain to assess their performance in:

- Predicting phase and amplitude at non-computed points. Note that FFT is used for temporal output to represent the result in the frequency domain.
- Extrapolating solutions beyond the dataset domain.

The networks are trained using the Adam optimizer with a learning rate of 0.001. A total of 12,000 training iterations are performed. MSE of the physics loss is scaled by 0.01 relative to MSE of the data loss to provide correct training.

RESULTS AND DISCUSSION

Approximation accuracy for different models

The approximation accuracy is tested for simple NN and PINN in the time and frequency domains. Figure 4 contains the mean square error for different NNs depending on the number of measurements per wavelength. In other words, there is a dependance of error on resolution. It is obvious that decreasing resolution increases error for all cases. However, PINNs approximate data better in the case of decreasing resolution. PINNs stop working if the sampling rate is less than the Nyquist rate (< 2 in Fig. 4).

Note that this result is valid for PINNs in time and frequency domains. So, PINNs can be used to increase resolution, which decreases the number of measurements.

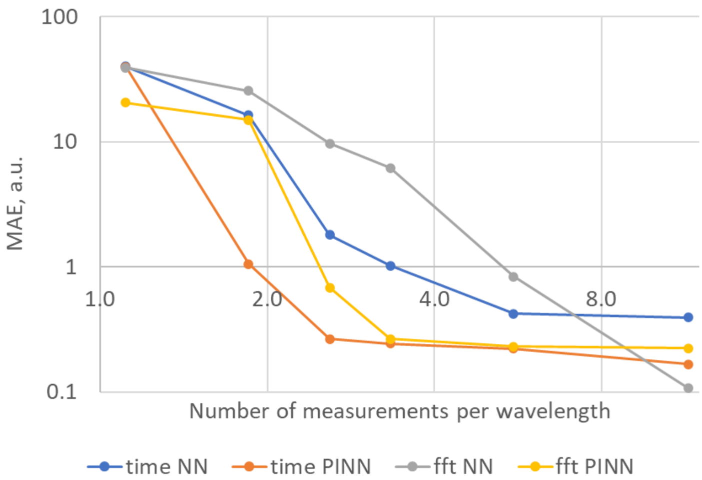

Fig. 4. Approximation error of different NNs depending on data density.

Extrapolation Capability

Ordinary NN and PINN are trained in limited regions and demonstrate different extrapolation performance in the frequency domain. Fig. 5 shows that an ordinary NN gives expectedly incorrect results for the single source.

On the other hand, Fig. 6 shows that PINNs can predict phase and amplitude in neighboring regions where phase is predicted with 20% error up to 0.5 of the training cell size, which is marked by a red rectangle. An exception is the neighboring region that is directed to the source. This is caused by the absence of the wave excitation source in the partial differential equation.

Using PINNs in the frequency domain for predicting narrow band signals allows for a reduction in computational resources.

Extrapolation of PINN in the time domain predicts phase correctly at a distance of 1 cell except the direction to the source (Fig. 7). Amplitude is predicted with an error of < 20% at a distance of 0.5 cells except the direction to the source. Large amplitudes cannot be correctly described by the model in both domains.

Also, Fig. 7 illustrates on the bottom images, predicted wave propagation in the time domain for two sources in opposite phase. It has a similar result to a single source. And these results are better than those for the frequency domain. This allows us to reduce the amount of data for training.

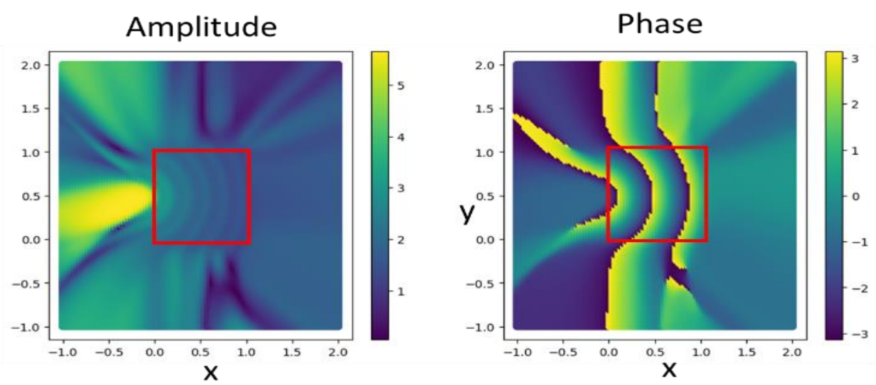

Fig. 5. Extrapolation results in frequency domain of ordinary NN for single source.

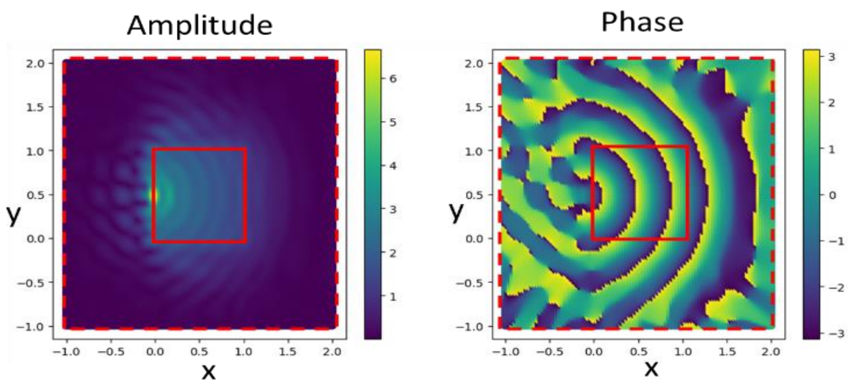

Fig. 6. Extrapolation results in frequency domain of PINN for single source.

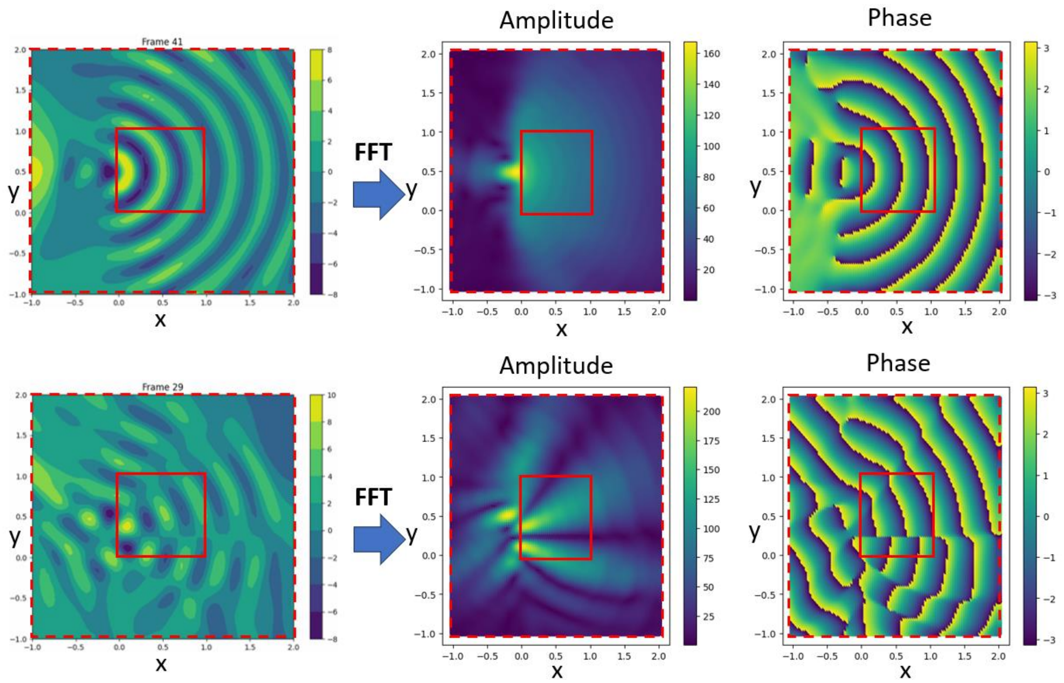

Fig. 7. Extrapolation results in time domain of PINN for single (top) and two (bottom) sources.

Despite the advantages of PINNs, it exhibits the following limitation. Nonlinearities in the neural network architecture prevent exact equivalence between time-domain and frequency-domain solutions.

CONCLUSION

PINNs offer a powerful tool for modeling wave propagation in scenarios where measurement data is limited in some areas. This study demonstrates their ability to improve resolution, extrapolate solutions, and reduce the need for additional data and computations. While challenges remain, particularly in the accurate prediction of large amplitudes and the preservation of solution identity across domains, PINNs represent a promising approach for applications in navigation and wireless tracking.

Future work should focus on improving the accuracy of predictions through better network architectures or hybrid approaches.

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ФІЗИЧНО-ІНФОРМОВАНІ НЕЙРОННІ МЕРЕЖІ ДЛЯ МОДЕЛЮВАННЯ ПОШИРЕННЯ ВУЗЬКОСМУГОВИХ СИГНАЛІВ

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АНОТАЦІЯ

Вступ. Фізично-інформовані нейронні мережі (ФІНМ) продемонстрували потужні можливості у вирішенні прямих і обернених задач для диференціальних рівнянь у часткових похідних. У цьому дослідженні основна увага приділяється застосуванню ФІНМ для апроксимації та екстраполяції поширення вузькосмугового сигналу. Ці зусилля мотивовані потенційною можливістю зменшити витрати на вимірювання та чисельні витрати в таких застосунках, як навігаційні системи на основі акустичних і електромагнітних маяків. Ці системи створюють карту середовища та відстежують траєкторії об'єктів, використовуючи дані про поширення хвиль.

Матеріали та методи. Поширення гармонійних хвиль у середовищі можна описати за допомогою хвильового рівняння або рівняння Гельмгольца. Для встановлення зв'язку між цими рівняннями використовується перетворення Фур'є. ФІНМ навчені в часовій або частотній області для прогнозування характеристик поширення хвилі, таких як амплітуда та фаза. Дослідження порівнює продуктивність ФІНМ зі звичайними нейронними мережами.

Результати. Дослідження показує, що ФІНМ демонструють кращу продуктивність у порівнянні зі звичайними нейронними мережами, коли точки тренувальних даних рознесені до частоти Найквіста. У часовій області ФІНМ точно передбачають фазу на відстані до однієї комірки, за винятком напрямку на джерело. Однак прогнози амплітуди менш точні, з помилками менше 20% на відстані до 0,5 клітини. Для більших амплітуд моделі важко забезпечити надійні прогнози. Навчання ФІНМ у частотній області потребує менше обчислювальних ресурсів, але продуктивність нижча, ніж у часовій області.

Висновки. ФІНМ пропонують багатообіцяючі переваги для моделювання розповсюдження хвиль у вузькосмугових сигналах, особливо в сценаріях, де дані вимірювань розріджені або локальні. Вони можуть збільшити роздільну здатність, зменшити обсяг необхідних даних і оптимізувати обчислювальну ефективність. Незважаючи на їх обмеження, існує різниця рішень між часовою та частотною областями через нелінійну природу нейронних мереж. Майбутня робота може стосуватися точності прогнозів за допомогою кращої архітектури мереж або гібридних підходів.

Ключові слова: Фізично-інформовані нейронні мережі, ФІНМ, хвилі, надвисока роздільна здатність, глибоке навчання, швидке перетворення Фур'є.

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