

INTERREGIONAL AND INTERTEMPORAL DIFFERENCES IN SPATIAL POLARIZATION AND RESILIENCE OF LOCAL BUDGET REVENUES IN UKRAINE UNDER MILITARY AGGRESSION

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Abstract. *The purpose of the article is to test the statistical relationship between resilience of territorial community budgets' real tax revenues and bimodality indicators for tax revenue distribution per capita during the period of armed aggression against Ukraine. Bimodality measurement was carried out using the method by Ashman et al., the maximum likelihood-based testing of hypothesis that the variable data set is generated by two or more processes, each having normal distribution of the variable. It uses an additional criterion (D-criterion), i.e. ratio of means difference to square root of average variance; with the criterion value above the threshold of 2, the maximum likelihood estimation favours bimodality model over unimodality. Application of bootstrapping technique allows both direct bimodality measurement and testing statistical hypotheses on the distribution shape, utilizing a number of criteria. For the direct relationship testing, Welch unequal variances t-test was used. The test results allow rejection of null hypothesis at a confidence level of more than 95%. At the same time, the D-criterion value difference between more and less resilient sets of communities lies in the 95% confidence interval from -0.55 to -1.01. Thus, more resilient sets are characterized by a lower degree of Ashman bimodality, which is the opposite result to the use of alternative polarization and bimodality indicators. Accordingly, further research requires theoretical analysis, in particular building a comprehensive economic-statistical and spatial-economic model for conducting a numerical simulation study.*

Keywords: *bimodality, Ashman's D, spatial structural disparities, spatial sustainability, resilience.*

Introduction. The formation of spatial policy for sustainable development is one aspect of the regulation of structural change in the national economy, including the mitigation of the consequences of armed aggression and the challenges of sustainable post-war recovery. The available body of work points towards the necessity of coordinating nation-level regulatory and institutional measures with the traditional domains of regional and local development policy. Identifying the conceptual basis for this kind of coordination would allow us to refine the concept of spatial sustainable development as the management of systemic functions in



ecosocioeconomic space (Hryniv, 2023). Accordingly, the problem of the interrelation between systemic sustainability and spatial structuring in the national and regional economy requires in-depth theoretical and applied research.

The phenomenon of economic development polarization, which is a symptom of economic structural disparities in both regional and national economies, statistically represents a distributional quality of spatially heterogeneous variables. The analytic methodology for spatial heterogeneity allows the application of traditional panel data analysis tools; however, a number of difficulties indicate the insufficiency of this toolkit. First, development polarization does not represent an objectively independent quality, but arises as a bound consequence of economic concentration and specialization. Second, the category of development polarization is still insufficiently refined, due to the conflation of the uneven spatial distribution of economic activity in a single system (e.g. the phenomenon of production and labour demand concentration) with the spatial-structural discontinuity of economic processes (e.g. the involvement and accumulation of human and physical capital) in isolated territorial communities. From the statistical perspective, development polarization in these two cases will manifest, respectively, in the forms of distribution kurtosis and bimodality. Theoretical and analytical separation of these scenarios has practical significance for public administration, as it allows the assessment and prevention of spatial sectoral disparities, of the disintegration of regional and national markets, and of the formation of a dual economy.

Spatial econometric methodology (Anselin, 2003) allows spatial structure to be considered a moderator, i.e. an additional exogenous factor, of systemic, naturally extra-spatial economic interrelations. However, structuring the problem along these lines only makes it possible to reveal the exogenous impact of spatial economic structures, but not to consider the regulation or improvement of these structures. Thus, the identification of methods for measuring spatial polarization, and the development of indicators suitable for assessing statistical effect sizes, are of primary methodological importance. This would facilitate building the appropriate models to test hypotheses concerning the effects of spatial economic organization on various levels of the national economy.

Literature review. Polarization quantification requires the identification and classification of indicators, testing for the statistical significance of their observed values, as well as of the values' temporal changes due to external interventions in particular – all of these being interrelated issues. One can identify a) indicators of structural inequalities, including the Gini coefficient and informational entropy indices, such as the Theil and Atkinson indices (Novotny, 2007); b) quantifiers of sample distribution shape, including the Pearson coefficient, the Ashman coefficient, and relative distance to the median (OECD, 2021). The generalized approach to testing for statistical significance is based upon the application of bootstrapping techniques.

A number of indicators from the second group allow the immediate measurement of bimodality, i.e. the degree of manifestation of two or more distribution density centres, which can be considered a consequence of the spatial disintegration of economic activity. The major theoretical results date back quite some time; the Pearson coefficient (i.e. the difference between distributional excess and squared kurtosis) is still the most informative.

It is equal to 1 for a Bernoulli distribution with two discrete values and tends toward infinity for unimodal distributions. However, like other indicators within the identified second group, the Pearson coefficient is conceptually detached from statistical hypothesis testing. On the other hand, the statistical test by Haldane (1951), based upon sample division into frequency categories, allows the sequential testing of hypotheses of bitangentiality and of statistically significant modes and antimodes among these categories. The method by Ashman et al. (1994) is the maximum likelihood-based testing of the hypothesis that the variable data set is generated by two or more processes, each having a normal distribution of the variable. It uses an additional criterion (D-criterion), i.e. the ratio of the difference in means to the square root of the average variance; with the criterion value above the threshold of 2, the maximum likelihood estimation favours a bimodality model over a unimodality model. In other words, a lower value of the indicator points to a lower degree of sample bimodality. The application of the bootstrapping technique as proposed by Muratov and Gnedin (2010) allows both the direct measurement of bimodality and the testing of statistical hypotheses on the distribution shape, utilizing a number of criteria.

The impact of the armed aggression against Ukraine, the ensuing warfare, and the post-deoccupation revival on the tax capacity of local administration in its spatial dimension is a practical issue to investigate using the development polarization analysis toolkit. Nakonechna and Demydenko document the sources and structural changes in local budget revenues under war conditions, while Voznyak and Sorokovyi (2025) identify the recent spatioregional structure of tax capacity, which necessitates the spatial and functional differentiation of regional development policy. At lower levels of aggregation, Vлах, Vanda and Kotyk (2022) explicate the spatial aspect of local budgets' tax capacity by proposing a multicriterial approach to estimating the integrated capacity of territorial communities as a sum of normalized spatial criteria that reflects the ability of polarization processes identified by separate variables to become mutually compensated. Both interregional and intraregional spatial differences in tax capacity structure point towards the necessary flexibility and place-based sensitivity of spatial development policies. Lysyak and Karyy (2025) identify, in particular, dedicated mechanisms for inter-municipal cooperation, territorial planning, and local business development among the required components of spatial development models for deoccupied communities. Baranovskyi (2026) suggests introducing new functional territorial types into the framework of the updated State Regional Development Strategy, conditioned upon both the spatial structure of developmental capacity and proximity to warfare areas Khodyko (2025) has previously applied a bootstrapping technique and specific sign testing to identify the statistical significance of the observed polarization of local per capita tax revenue, as well as of its intertemporal change, based on the polarization coefficient designed by OECD (2021).

Aims. The purpose of the article is to preliminarily test the statistical relationship between the resilience of territorial community budgets' real tax revenues and bimodality indicators for tax revenue distribution per capita during the period of armed aggression against Ukraine. Bimodality measurement was carried out using the method by Ashman et al. (1994) with modifications by Muratov and Gnedin (2010). For direct testing, the Welch unequal-variances t-test (1947) was used. The test results allow rejection of the null hypothesis at a confidence

level of more than 95% for the sample of four regions (with a sample size of 235 communities), including deoccupied communities as well as those located both remotely from and in direct proximity to the warfare area.

Methods and Results. The possibility of developing statistical hypotheses and models is warranted by the availability of data, if still relatively scarce at present, on local budget revenues in the newly established territorial communities, predominantly founded in 2019–2020 (Kaziuk, 2023; State Tax Service, 2025). Table 1 below summarizes the data on the revenues and their temporal dynamics at the regional level. Two of the deoccupied Ukrainian regions, Sumy and Chernihiv, demonstrated quite similar values, both in region-level local budget tax revenues and in their community-level spatial per capita distribution polarization, as of 2020. These indicators differed considerably in 2023. In real terms, Sumy region showed a decrease in total tax revenues of 7.9% from the 2021 level; Chernihiv region, a decrease of only 1.5%. Per capita tax revenues were more polarized in Chernihiv region, which was more resilient against external threats. Tax maps of Ukraine, based on State Tax Service data, showcase a high revenue centre consisting of several neighbouring communities in Chernihiv, in contrast to a number of such centres in Sumy region. There is no apparent relationship between this spatial distribution and the deoccupation maps of the regions. Tax revenue polarization by the Pearson and OECD criteria declined drastically in 2024, as did the absolute revenue amounts; both regions demonstrated a real revenue decline of 24% against the base year 2021, accompanied by the disappearance of the spatial centres seen previously. The deoccupied Kharkiv region showed a real reduction of total revenues of 44% in 2023, and of 52% in 2024, compared to 2021. For Kharkiv region, as for the other deoccupied regions, the Pearson coefficient indicated an increase in polarization in 2023, followed by a decline in 2024, against the year 2021.

Table 1

**Levels and dynamics of territorial communities' budget revenues
in Ukrainian regions (2021–2024)**

Region	Sumy	Chernihiv	Kharkiv	Lviv
Tax revenues in 2021, in hryvnias	6957929914	6709113740	21874442378	17549005536
Tax revenues in 2021 (2023 prices), UAH	9125227671	8838479095	28395388647	22825219344
Consumer price index (December 2021–September 2023)	1.3115	1.3174	1.2981	1.3007
Tax revenues in 2023 (estimate based on three quarters), UAH	8407573376	8710333396	15777479764	26812832469
Changes in real tax revenues, 2021–2023 (estimate)	-7.86%	-1.45%	-44.44%	17.47%
Tax revenues in 2021 (2024 prices), in hryvnias	9849252088	9656593095	30788305223	24681852167
Consumer price index (December 2021–September 2024)	1.4155	1.4393	1.4075	1.4065
Tax revenues in 2024 (estimate based on three quarters), UAH	7503511724	7325846223	14677071563	25042095601
Changes in real tax revenues, 2021–2024 (estimate)	-23.82%	-24.14%	-52.33%	1.46%

Sources: [15], [16], author's calculations.

By contrast, Lviv region is distinguished by a historically higher degree of tax revenue polarization. It demonstrated an increase in real per capita revenues of 17.5% in 2023 against 2021. The real revenues in 2024 were higher by 1.5% compared to the base year. The general reduction in tax capacity, similarly to the deoccupied regions, is accompanied by a decline in its polarization according to both the Pearson and OECD indicators (Khodyko, 2025). Accordingly, the methodology for choosing polarization indicators suitable for advancing and testing formal statistical hypotheses gains relevance.

As the Haldane test results suggest, the bimodality of tax revenue distribution in the samples considered accompanies the region's higher resilience against external factors and threats. Notably, the result stems from the evaluation of sample data and does not represent hypothesis testing on the links between regional resilience and spatial polarization phenomena. A more important reservation is that the Haldane test compares alternative hypotheses and is not capable of quantifying hypothetical effect sizes. Therefore, the issue of choosing a methodology for quantifying spatial distribution bimodality is still open.

Table 2

Levels and dynamics of budget revenues in deoccupied and non-occupied
Ukrainian territorial communities by region (2021–2024)

deoccupied territorial communities, by region	Sumy	Chernihiv	Kharkiv
Tax revenues in 2021, UAH	2225581446	1872485653	2820509456
Tax revenues in 2021 (2023 prices), UAH	2918818908	2466782639	3661325889
Consumer price index (December 2021–September 2023)	1.3115	1.3174	1.2981
Tax revenues in 2023 (estimate based on three quarters), UAH	2959090740	3165132009	1843904077
Changes in real tax revenues, 2021–2023 (estimate)	1.38%	28.31%	-49.64%
Tax revenues in 2021 (2024 prices), UAH	3150407230	2695114843	3969870615
Consumer price index (December 2021–September 2024)	1.4155	1.4393	1.4075
Tax revenues in 2024 (estimate based on three quarters), UAH	2176735195	1920740829	1443007838
Changes in real tax revenues, 2021–2024 (estimate)	-30.91%	-28.73%	-63.65%
territorial communities never occupied, by region	Sumy	Chernihiv	Kharkiv
Tax revenues in 2021, UAH	4732348468	4836628087	18921981974
Tax revenues in 2021 (2023 prices), UAH	6206408762	6371696456	24562776176
Consumer price index (December 2021–September 2023)	1.3115	1.3174	1.2981
Tax revenues in 2023 (estimate based on three quarters), UAH	5448482636	5545201387	13922415380
Changes in real tax revenues, 2021–2023 (estimate)	-12.21%	-12.97%	-43.32%
Tax revenues in 2021 (2024 prices), UAH	6698844858	6961478252	26632713481
Consumer price index (December 2021–September 2024)	1.4155	1.4393	1.4075
Tax revenues in 2024 (estimate based on three quarters), UAH	5326776529	5405105393	13223522001
Changes in real tax revenues, 2021–2024 (estimate)	-20.48%	-22.36%	-50.35%

Sources: [15], [16], author's calculations.

This warrants turning towards a component of the methodology by Ashman et al. (1994) concerned with the D-criterion for bimodality, as modified by Muratov and Gnedin (2010). A data set was used for four regions of Ukraine, including three deoccupied in 2022, namely Chernihiv, Sumy and Kharkiv, with the latter still belonging to the warfare area throughout

the time range considered, as well as Lviv region, located remotely from the warfare zone and constituting the leading destination for forced migration and production relocation. The D-criterion was estimated for each region based on accumulated amounts over three quarters in 2021, 2023 and 2024. Limiting the sample to three quarters is necessary for the comparability of the data, given the end-2023 resurgence of warfare in the Sumy and Kharkiv regions. In addition, subsamples of deoccupied communities, as well as those never occupied, were identified for each of the four regions; Table 2 above summarizes the tax revenue dynamics within the identified subsamples. No autocorrelation of data values within subsamples and regions was assumed, due to the random (with respect to per capita tax revenues) division into deoccupied and non-occupied communities.

Table 3

**Bimodality hypothesis tests and Ashman's D-criterion estimation
for local budgets' per capita tax revenues distribution in territorial communities
of Sumy and Chernihiv regions, Ukraine**

Region	Sumy			Chernihiv		
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>
Year	2021	2023	2024	2021	2023	2024
N	51	51	51	57	57	57
Sample						
Unimodal peak	6465.55	6607.18	5163.39	6366.71	7135.46	5145.42
Unimodal sigma	2250.50	4107.51	2351.11	1927.66	9332.03	1573.58
logL	-466.03	-496.72	-468.26	-512.03	-601.93	-500.46
Peak 1	6033.82	4695.78	4686.38	6213.74	5491.95	3654.50
Sigma 1	1582.67	1265.32	1394.55	1564.75	5589.49	389.06
Peak 2	11049.47	12072.83	11993.61	14933.10	43467.87	5821.18
Sigma 2	3041.98	4496.41	2630.91	391.19	22357.94	1437.25
logL	-459.37	-474.57	-454.64	-503.29	-562.56	-493.73
Chi ²	13.32	44.28	27.27	17.48	78.74	13.46
P	0.010	0.000	0.000	0.002	0.000	0.009
Ashman's D	2.07	2.23	3.47	7.65	2.33	2.06
Bootstrap						
Unimodal peak	6439.47	6561.23	5136.53	6349.85	7138.67	5140.84
Confidence interval (99.9%)	320.46	577.00	336.40	289.31	1315.86	241.26
Unimodal sigma	2214.64	4011.86	2295.84	1875.96	8297.56	1561.60
Confidence interval (99.9%)	368.23	631.72	478.75	338.87	3898.84	180.46
Peak 1	4930.31	4718.54	4404.47	5086.13	5295.19	3730.01
Confidence interval (99.9%)	866.98	265.71	438.07	802.00	437.35	310.12
Sigma 1	907.05	1291.35	1167.66	826.69	3496.80	454.30
Confidence interval (99.9%)	456.62	156.44	362.23	410.03	1831.84	184.37
Peak 2	9090.23	12619.33	10625.68	8470.94	39109.12	6060.23
Confidence interval (99.9%)	3045.34	1695.99	3284.82	2658.48	16021.67	751.20
Sigma 2	1934.65	3989.97	1822.91	1742.23	12261.96	1352.61

End of table 3

<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>
Confidence interval (99.9%)	709.33	952.59	992.66	826.24	9305.46	295.75
Ashman's D	2.78	2.81	4.26	2.64	9.13	2.33
Confidence interval (99.9%)	1.88	1.00	2.62	2.08	13.58	0.68
p(chi^2)	0.006	<0.001	<0.001	<0.001	<0.001	0.008
p(D)	0.688	0.618	0.167	<0.001	0.538	0.639

Sources: [15], [16], author's calculations.

Ashman's D criterion was therefore estimated for three samples in each of the four regions over three years, which produces 36 aggregated observations. In total, 237 territorial communities were included, except for two communities in Kharkiv region occupied in 2025, with no less than 21 communities (i.e. the number of deoccupied communities in Kharkiv region) per sample.

Table 4

Bimodality hypothesis tests and Ashman's D-criterion estimation for local budgets' per capita tax revenues distribution in territorial communities of Kharkiv and Lviv regions, Ukraine

Region	Lviv			Kharkiv		
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>
Year	2021	2023	2024	2021	2023	2024
N	73	73	73	56	54	54
Sample						
Unimodal peak	5140.15	6057.06	5331.63	6753.76	4502.89	4755.89
Unimodal sigma	2894.07	5125.05	3365.86	3304.45	3027.33	2895.80
logL	-685.42	-727.14	-694.45	-533.23	-509.46	-507.06
Peak 1	4404.06	4150.40	4592.60	5961.44	3267.76	4023.54
Sigma 1	1525.19	1773.52	1795.85	1588.22	1534.67	1702.46
Peak 2	10688.61	12768.28	12429.38	17003.01	8220.33	10841.49
Sigma 2	4370.96	7094.06	5791.05	2524.26	3353.91	3534.43
logL	-664.37	-694.47	-672.29	-508.41	-499.51	-495.56
Chi^2	42.10	65.34	48.32	49.65	19.90	22.99
P	0.000	0.000	0.000	0.000	0.001	0.000
Ashman's D	1.92	1.67	1.83	5.24	1.90	2.46
Bootstrap						
Unimodal peak	5158.76	6078.69	5353.74	6720.87	4471.63	4715.80
Confidence interval (99.9%)	396.91	685.73	456.18	518.56	486.72	465.52
Unimodal sigma	2878.08	5003.03	3313.22	3161.14	2973.90	2798.22
Confidence interval (99.9%)	547.67	1043.31	711.58	753.60	423.80	531.56
Peak 1	4397.86	4121.58	4409.86	5831.72	3252.94	3772.21
Confidence interval (99.9%)	233.28	482.22	418.87	497.94	522.13	680.03
Sigma 1	1525.35	1771.36	1621.84	1511.22	1492.72	1522.78

End of table 4

<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>
Confidence interval (99.9%)	159.40	433.63	332.68	252.35	320.91	370.18
Peak 2	11715.36	13956.20	12945.58	16203.59	9508.66	10891.50
Confidence interval (99.9%)	2475.85	4177.52	4529.45	2969.68	2497.99	2860.68
Sigma 2	3303.76	6383.66	3975.08	1719.33	2384.20	2247.63
Confidence interval (99.9%)	1425.04	1989.31	1661.77	1123.72	978.61	1286.03
Ashman's D	3.27	2.22	3.22	6.50	3.36	3.92
Confidence interval (99.9%)	2.28	1.38	2.73	2.45	1.86	2.02
p(chi ²)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
p(D)	0.648	0.720	0.684	0.006	0.702	0.524

Sources: [15], [16], author's calculations.

Calculations were performed using the dedicated software authored by Gnedin. Tables 3 to 6 present full results of bimodality hypothesis testing by maximum likelihood and by bootstrapping technique, as well as sample values and confidence intervals estimations of D-criterion for the regions and subsamples considered.

Table 5
Bimodality hypothesis tests and Ashman's D-criterion estimation for local budgets' per capita tax revenues distribution in deoccupied territorial communities in Sumy, Chernihiv, Kharkiv regions, Ukraine

Region	Sumy			Chernihiv			Kharkiv		
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>
Year	2021	2023	2024	2021	2023	2024	2021	2023	2024
N	26	26	26	27	27	27	21	21	21
Sample									
Unimodal peak	6247.3	6804.9	4451.9	6920.7	9684.5	5159.3	5767.8	3265.1	3113.0
Unimodal sigma	1373.3	4568.0	1368.4	2254.4	12991.2	1663.8	1274.5	2766.9	1372.6
logL	-223.7	-256.0	-224.6	-246.8	-294.1	-238.6	-180.0	-196.2	-181.5
Peak 1	4529.4	4648.0	3543.5	6612.6	6204.5	5238.6	5382.6	2302.8	1582.7
Sigma 1	296.2	1194.8	785.8	1647.4	5625.3	1432.7	1030.5	996.9	333.5
Peak 2	6689.8	13578.7	5782.3	14933.1	41945.3	5238.6	7621.5	7540.8	3677.6
Sigma 2	1184.7	4779.2	855.5	411.8	22501.1	2013.1	424.8	3854.6	1164.2
logL	-222.0	-238.8	-223.7	-240.7	-272.8	-238.6	-179.0	-186.8	-179.4
Chi ²	5.53	34.30	1.87	12.16	42.51	3.90	1.86	18.89	4.31
P	0.237	0.000	0.760	0.016	0.000	0.142	0.762	0.001	0.366
Ashman's D	2.50	2.56	2.73	6.93	2.18	2.59	2.84	1.86	2.45
Bootstrap									
Unimodal peak	6221.3	6796.1	4418.6	6914.3	9920.3	5132.6	5757.4	3332.1	3108.7
Confidence interval (99.9%)	266.0	915.3	264.8	453.3	2756.9	320.2	315.1	649.2	337.1
Unimodal sigma	1361.1	4478.7	1361.1	2232.2	12303.5	1644.7	1255.8	2728.7	1346.8

End of table 5

<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>
Confidence interval (99.9%)	152.9	1074.0	175.1	527.7	5494.8	264.6	147.5	831.0	176.2
Peak 1	4684.6	4631.9	3570.2	5871.8	5989.7	3932.9	4981.9	2157.7	2142.0
Confidence interval (99.9%)	406.3	211.8	301.2	623.4	547.1	232.2	570.1	436.4	626.4
Sigma 1	359.4	1110.0	753.4	1041.0	3771.2	500.4	773.6	897.2	610.9
Confidence interval (99.9%)	192.4	187.4	269.4	389.0	1846.7	165.6	266.2	287.6	319.2
Peak 2	6961.2	14363.5	5980.8	10536.7	43414.3	6658.6	7293.2	8299.2	4618.3
Confidence interval (99.9%)	592.7	2626.4	569.4	2744.5	16769.2	756.2	615.1	2989.9	967.3
Sigma 2	1055.8	3914.4	712.1	1677.7	13487.3	1310.6	494.1	2634.6	732.9
Confidence interval (99.9%)	261.3	1364.1	280.1	1111.2	9324.2	422.4	270.9	1563.9	412.9
Ashman's D	2.87	3.91	3.24	3.61	10.80	2.90	3.42	4.28	3.41
Confidence interval (99.9%)	0.55	3.00	0.78	2.49	17.68	1.10	0.78	4.41	1.02
p(chi ²)	0.227	<0.001	0.799	0.017	<0.001	0.028	0.779	<0.001	0.367
p(D)	0.672	0.653	0.590	<0.001	0.767	0.854	0.587	0.850	0.718

Sources: [15], [16], author's calculations.

To conduct the Welch test, the obtained D-criterion values were divided into two categories. The first included regional and subregional samples of communities which showed small declines or growth in real tax revenues for the sample as a whole, as specified in Table 2 above, namely the deoccupied communities in Chernihiv and Sumy in 2023, as well as all of Chernihiv and Lviv in 2023, and all of Lviv in 2024. Other samples were included in the second category. The Welch test for unequal variances is a modification of Student's t-test with a number of assumptions relaxed[14]. The test was conducted with two lists within the categories, either including or excluding the data for Lviv region, i.e. the second list considered only the regions belonging to the warfare area. Finally, the D-criterion values for the test were chosen based on maximum likelihood estimation in one version, while in the other, on central values of the bootstrap-estimated confidence interval. Table 7 below presents the categorical descriptive statistics and full Welch test results for the two lists and the two indicators.

Table 6

Bimodality hypothesis tests and Ashman's D-criterion estimation for local budgets' per capita tax revenues distribution in territorial communities never occupied in the Sumy, Chernihiv and Kharkiv regions, Ukraine

Region	Sumy			Chernihiv			Kharkiv		
Year	2021	2023	2024	2021	2023	2024	2021	2023	2024
N	25	25	25	30	30	30	35	33	33
Sample									
Unimodal peak	6692.5	6401.6	5903.3	5868.1	4841.3	5133.0	7345.3	5290.5	5801.4
Unimodal sigma	2875.7	3554.5	2873.2	1400.4	1568.3	1487.6	3945.0	2920.6	3116.5
logL	-234.6	-239.9	-234.6	-259.9	-263.3	-261.7	339.5	-310.1	-312.3
Peak 1	5225.0	5041.6	4964.6	4028.5	4196.4	3469.9	6092.7	4068.2	4751.8
Sigma 1	1353.7	1731.2	1369.7	351.6	1036.6	326.4	1757.4	1573.7	1615.8
Peak 2	9187.9	13095.0	11973.7	6257.8	7244.4	5676.2	17000.2	9016.8	11225.5
Sigma 2	3053.8	2482.5	2637.7	1217.3	544.4	1305.8	2524.6	2922.8	3361.3
logL	-231.1	-233.5	-226.8	-258.3	-260.4	-258.6	-325.0	-305.4	-305.0
Chi ²	6.91	12.76	15.48	3.14	5.81	6.14	28.97	9.51	14.64
P	0.141	0.013	0.004	0.535	0.214	0.189	0.000	0.050	0.006
Ashman's D	1.68	3.76	3.34	2.49	3.23	2.32	5.02	2.11	2.45
Bootstrap									
Unimodal peak	6686.5	6384.6	5915.6	5840.7	4810.5	5120.2	7314.4	5255.2	5775.1
Confidence interval (99.9%)	643.0	799.1	650.8	270.2	303.2	284.9	706.9	511.8	552.6
Unimodal sigma	2845.7	3487.3	2824.6	1390.3	1543.2	1479.7	3831.1	2902.8	3089.8
Confidence interval (99.9%)	553.7	657.2	689.2	147.6	152.0	230.7	804.2	466.4	628.5
Peak 1	4955.6	4411.2	4599.6	4427.4	404.6	4170.5	5851.8	3971.1	4563.9
Confidence interval (99.9%)	780.2	812.6	535.5	577.8	486.3	737.8	647.1	474.4	586.0
Sigma 1	1135.2	1290.4	1097.5	534.5	928.1	732.9	1613.2	1443.7	1493.9
Confidence interval (99.9%)	393.9	410.2	343.8	281.8	257.9	416.1	336.3	397.4	321.3
Peak 2	10064.1	11310.6	10736.0	6816.9	7106.4	7400.3	16019.4	9816.0	11659.1
Confidence interval (99.9%)	2451.1	3023.3	3004.5	742.4	710.8	1876.9	3110.9	2272.2	2636.4
Sigma 2	2204.1	2344.7	2054.3	952.9	538.0	723.1	2061.3	2027.5	2344.3
Confidence interval (99.9%)	1010.0	1113.6	970.4	313.2	289.9	488.4	1142.3	960.8	1263.6
Ashman's D	3.07	4.00	4.02	3.01	3.89	3.98	5.64	3.49	3.89
Confidence interval (99.9%)	1.77	2.35	2.55	0.68	0.69	1.46	2.18	1.84	2.17
p(chi ²)	0.152	0.009	0.002	0.560	0.193	0.169	<0.001	0.038	0.002
p(D)	0.869	0.199	0.345	0.641	0.164	0.699	0.022	0.724	0.610

Sources: [15], [16], author's calculations.

In the case of maximum likelihood values for D-criterion, the test results in Table 7 allow rejection of the two-tailed null hypothesis both for the four-region dataset (including Lviv) and for the three-region one (deoccupied regions only) at the 95% confidence level. The respective Student's t-criterion values are equal to 2.238 with $df=12$ ($p=0.0449$) and 2.906 ($df=12.45 \sim 12$, $p=0.0132$). The difference in D-criterion values between the more and less resilient community samples lies within the 95% confidence interval of -0.55 to -1.01 (four regions) and -0.75 to -1.20 (three deoccupied regions). Therefore, resilient samples demonstrate a lower degree of Ashman bimodality, which is the opposite trend to that found when using the Pearson bimodality and OECD polarization coefficients. The D-criterion values estimated within the bootstrapping confidence interval do not allow rejection of the null hypothesis.

Table 7

**Welch test for interregional and intertemporal difference
 in Ashman's D polarization of budget revenues**

	Deoccupied regions				Deoccupied + Lviv region			
	Maximum likelihood		Bootstrap estimation		Maximum likelihood		Bootstrap estimation	
	resilient	non-resilient	resilient	non-resilient	resilient	non-resilient	resilient	non-resilient
Number of observations	3	24	3	24	5	25	5	25
Mean D-criterion	2.36	3.14	7.95	3.64	2.11	3.09	5.86	3.62
corrected standard deviation	0.0902	0.3386	1.6943	0.2400	0.1453	0.3036	1.5378	0.1792
D-criterion difference	-0.78		4.31		-0.98		2.23	
t-statistic	2.2378		2.5177		2.9057		1.4420	
Df	12.0		2.0		12.5		4.0	
p (two-tailed)	0.0450		0.1281		0.0132		0.2228	
D-criterion difference CI (+95%)	-0.55				-0.76			
D-criterion difference CI (-95%)	-1.01				-1.20			

Sources: [15], [16], author's calculations.

As an additional result, one can note the high D-criterion values for Chernihiv region in 2021, both as a whole and in the deoccupied sample, which even surpass the relatively high value for the less resilient Kharkiv region. This raises the question of statistical links between bimodality indicator values and their temporal changes, on the one hand, and the region's resilience potential and its actualization in the case of external threats, on the other. A theoretical interpretation of the obtained quantitative results appears necessary at this stage.

Conclusions. The toolkit applied is the one most directly related to the outlined theoretical and methodological issues and has advantages over both the Haldane test and the Pearson coefficient, due to the possibility of direct bimodality measurement and hypothesis testing on its dynamics. Further research, however, requires theoretical analysis for a number of reasons. First, the obtained

results, indicating the opposite direction of the tested statistical links and sample values for alternative polarization and bimodality criteria, demand comparison of the respective statistical distribution shapes to ensure the logical consistency of the results. Second, observational data points with both present and absent statistical significance for D-criterion confidence intervals were used for the Welch test conducted. Simulation research to identify the statistical power of the applied tests and regression models under different levels of statistical significance of measurement results can be noted as one of the potential methods for clarifying the methodological effectiveness of our approach. Third, the expected impacts of spatial economic concentration, polarization, and sectoral disparity on the values and dynamics of a wide range of spatial structural indicators, at both regional and national levels, need to be explicated theoretically.

The above reasons necessitate building a complex statistical and spatial economic model for numerical simulation research concerning the noted issues, which defines a number of tasks for future research.

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МІЖРЕГІОНАЛЬНІ І МІЖЧАСОВІ ВІДМІННОСТІ ПРОСТОРОВОЇ ПОЛЯРИЗАЦІЇ ТА ОПІРНОСТІ ДОХОДІВ МІСЦЕВИХ БЮДЖЕТІВ В УКРАЇНІ В УМОВАХ ВІЙСЬКОВОЇ АГРЕСІЇ

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Анотація. Формування просторової політики сталого розвитку є одним із аспектів державного регулювання структурних змін у національній економіці, зокрема з точки зору подолання наслідків військової агресії проти України та післявоєнного відновлення на засадах сталого розвитку. Поглибленого теоретичного й прикладного дослідження потребує проблема співвідношення системної стійкості та просторової структурованості в національній і регіональній економіці. Зокрема, категорія поляризації є недостатньо розробленою, оскільки включає в себе як нерівномірність просторового розподілу економічної діяльності, так і просторово-структурну розірваність економічних процесів у територіальних громадах. Важливого методичного значення набувають питання визначення способів вимірювання просторової поляризації та розробка показників, придатних до оцінки сили ефекту у статистичних моделях.

Метою статті є тестування статистичного зв'язку між стійкістю податкових надходжень до бюджетів територіальних громад в період збройної агресії проти України та статистичними характеристиками їх розподілу. Емпіричною базою дослідження є набори даних для трьох

прифронтових регіонів України (Чернігівської, Сумської та Харківської), а також Львівської області. Сумська та Чернігівська області України у 2020 році демонстрували доволі подібні результати як за загальним обсягом податкових надходжень до місцевих бюджетів, так і за поляризацією їх просторового розподілу на рівні територіальних громад. Сумська область продемонструвала зниження податкових надходжень на 7,9% порівняно з обсягом 2021 року; Чернігівська область – зниження лише на 1,5%. Харківська область у 2023 році продемонструвала реальне скорочення загальних надходжень на 44%, у 2024 році – на 52% порівняно із 2021 роком. Податкові надходження на особу у стійкіших регіонах були більш поляризованими, а скорочення абсолютних реальних надходжень супроводжувалося зниженням бімодальності за Пірсоном. Використання бутстреп-методики дає змогу як здійснювати безпосереднє вимірювання ступеня бімодальності, так і тестувати статистичні гіпотези щодо форми розподілу за рядом критеріїв. Оцінювання D-критерію Ешмана як індикатора бімодальності проводилося для кожної області упродовж трьох кварталів 2021, 2023 і 2024 років. Тестування гіпотези здійснювалося із застосуванням t-тесту Велча для вибірок з нерівною дисперсією. Результати тестування дозволяють відкинути нуль-гіпотезу за рівня довіри більше 95%. Водночас різниця у значеннях D-критерію між вибірками громад, які відрізняються за рівне стійкості, лежить у 95-відсотковому довірчому інтервалі між -0,55 та -1,01, тобто, вибірки громад, що показують вищу резилієнтність податкових надходжень, характеризуються нижчим ступенем бімодальності за Ешманом. Цей результат є протилежним до застосування коефіцієнту бімодальності Пірсона та коефіцієнту поляризації за методикою OECD. Виходячи з цього, перспективи подальших досліджень пов'язані в першу чергу із теоретичною інтерпретацією одержаних кількісних результатів.

Ключові слова: бімодальність, D-критерій Ешмана, просторово-структурні диспропорції, просторова стійкість, опірність, резилієнтність.

Стаття: надійшла до редакції 25.04.2026

прийнята до друку 25.06.2026

опублікована (оприлюднена) 30.06.2026