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THE METHOD OF EVALUATING THERMAL PHYSICAL CHARACTERISTICS OF BUILDINGS BASED ON THE INVERSE PROBLEM OF THERMAL CONDUCTIVITY

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The paper describes the method of calculating the effective thermophysical parameters of the building based on the analysis of changes in external and internal temperatures during the cold period of the year, as well as data on energy consumption during heating. Here, the finite difference method is used to solve a direct problem, which consists in determining the temperature regime in the building, taking into account the relevant thermophysical properties of materials. The paper also offers a numerical scheme for calculating the steps of the method and investigates the influence of thermophysical parameters on the calculated temperature values. Based on the results of the direct problem, the inverse problem of identifying the effective thermal parameters of the building is formulated and solved. To solve this problem, the methods of rough sorting are used to obtain an approximate solution and the quasi-Newton BFGS algorithm is used for its further refinement.

Key words: thermophysical modeling, direct and inverse problems of thermal conductivity, numerical optimization.

Introduction. In parallel with the development of systems for managing the thermal regime of the house, the creation of various models for their improvement and the introduction of predictive models of climatic indicators, it is important to determine the thermophysical characteristics of the building. These characteristics include both the effective heat capacity and the thermal conductivity of the building walls. Information on these parameters, as well as data on consumed energy, temperature regime and settings of thermoregulating devices, can be used as a basis for creating highly efficient energy consumption management systems.

Among various approaches to modeling the building's thermal behavior, such as the "white", "black" and "gray" box methods, the latter combines the statistical and physical essence of the building's thermal parameters [1]. Thus, the main purpose of this method is to model both heat input and heat loss of the building. For example, in [2] data on temperature and energy were used in differential equations for modeling the thermal behavior of the house based on the "gray" box method. In [3], a method for calculating the coefficient of heat loss of the house based on short-term monitoring data was proposed, using two approaches: the "gray box" method and statistical modeling. Another contribution to these methods was made in [4], where the authors used heat transfer equations to establish the relationship between consumed and lost energy, as well as the number of degree-days for increasing the temperature in the room. In order to improve the existing methods, new metrics were developed in [1] for assessing the effect of thermal mass on the required energy for heating and cooling buildings.

This paper considers the problem of identifying the thermophysical parameters of the building based on the solutions of the formulated direct and inverse problems. A technique for their solution based on a combination of finite difference and optimization methods is proposed. The obtained results can be useful for creating more accurate models of thermal behavior and optimizing the operation of heat preservation systems in buildings. The methodology and results presented in the paper are based on the materials of the dissertation [5].

Motivation and problem statement. Usually, when designing climate systems for the house, measurements (temperature, humidity, energy consumption, etc.) are used, which are analyzed using statistical methods. After that, this data can be used to model and create intelligent systems. Despite the efficiency of these systems, it is often useful to include physical equations in the developed models. These equations not only describe the thermal behavior in the room, but can also be informatively integrated with statistical data analysis algorithms.

For example, knowledge about the characteristics of the walls, such as their heat capacity and thermal conductivity, allows you to adjust the heating parameters in individual rooms and use this data in the development of a smart thermostat. These characteristics can be determined by solving the corresponding heat conduction equations. However, since the heat conduction equation contains unknown parameters (or those whose exact values are difficult to determine), for example, coefficients of thermal conductivity and heat capacity, there is a need to formulate inverse problems to estimate these unknown characteristics.

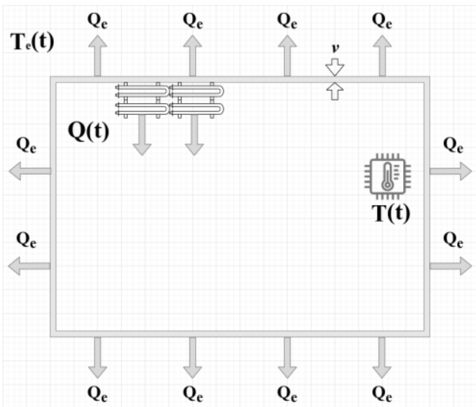

Fig. 1. Scheme of the thermal process in the building

Consider a building (Fig. 1) equipped with internal and external temperature sensors, as well as a meter of consumed heating energy. Not only the known physical characteristics of the walls and insulation, but also the results of solving the heat conduction equation can be used to estimate the complex thermophysical properties of the building. These parameters include effective coefficients of thermal conductivity and heat capacity. To perform modeling and analysis of the proposed approach, the article used data obtained from the REFIT open database [6]: distributions of external and averaged internal temperatures, as well as total gas consumption as a source of building heating.

The differential heat transfer equation applied to the building heating process (Fig. 1, where Q_e — is the external heat flow from the walls of the building) can be presented in the following form:

$$c(\boldsymbol{r})\rho(\boldsymbol{r})\frac{\partial T(\boldsymbol{r},t)}{\partial t} = div\left(\kappa(\boldsymbol{r})\nabla T(\boldsymbol{r},t)\right) + Q(\boldsymbol{r},t), \tag{1}$$

where $c(\boldsymbol{r})$ — specific heat capacity, $\rho(\boldsymbol{r})$ — density of the medium in which heat spreads, $\boldsymbol{r} = \boldsymbol{r}(x_1, x_2, x_3)$ — radius vector of a point in space, $\kappa(\boldsymbol{r})$ — coefficient of thermal conductivity, $T(\boldsymbol{r},t)$ — temperature in the room, $Q(\boldsymbol{r},t)$ — local value of the thermal power released in the house.

Equation (1) must be supplemented with boundary conditions that describe the heat flow through the surface of the selected volume.

$$-\kappa\frac{\partial T(\boldsymbol{r},t)}{\partial \boldsymbol{n}} = \alpha\left(T_e - T\right), \tag{2}$$

where $\boldsymbol{n}$ — external normal to the surface, α — heat transfer coefficient, T_e — the temperature of the outside of the selected volume.

By integrating equation (1) over the volume of the house V and using the Ostrogradsky-Gauss theorem, we obtain the following differential equation for the average temperature in the room $\bar{T}(t)$:

$$c^*\frac{\partial \bar{T}(t)}{\partial t} = -\kappa^*\left(\bar{T}(t) - T_e(t)\right) + \bar{Q}(t), \tag{3}$$

where c^* [J/(m^{3}K)] — effective (averaged) volumetric heat capacity, κ^* [W/(m^{3}K)] — normalized by V the heat transfer coefficient of the external walls of the building, which is proportional to their area and thermal conductivity coefficient and inversely proportional to the average wall thickness, $\bar{Q}(t)$ — the average power that is released through 1 m^3 of the house.

Note that equation (3) can be easily obtained by considering the heat balance in the house. Entering the temperature difference $T^*(t) = \bar{T}(t) - T_e(t)$, let's write equation (3) in a form convenient for further numerical solution

$$c^*\frac{\partial T^*(t)}{\partial t} = -\kappa^* T^*(t) + \bar{Q}(t) - c^*\frac{\partial T_e(t)}{\partial t}. \tag{4}$$

Equation (4) can be used to calculate the dynamics of temperature changes in the house, as well as to find effective coefficients c^* and κ^* (inverse problem).

To solve equation (4), we discretize it by time, which allows us to use real temperature and power measurements to study the ranges of values of the effective heat capacity parameters c^* and κ^*.

The discrete representation of equation (4), constructed using the finite difference method, has the following form:

$$c^* \frac{T^*(t_i) - T^*(t_{i-1})}{t_i - t_{i-1}} = -\kappa^* T^*(t_i) + \bar{Q}(t_i) - c^* \frac{T_e(t_i) - T_e(t_{i-1})}{t_i - t_{i-1}}, \qquad (5)$$

where t_i denotes successive moments of time. Equation (5) is supplemented by the initial condition of the known temperature in the room t_0^* in moment of time t_0: $T^*(t_0) = t_0^*$.

Here we assume that $\bar{Q}(t)$ and $T_e(t)$ are known functions defined for $t > t_0$. The above-described model defines a direct problem of the thermal state of the house, which will be used in the future to study the distributions of the parameters of the effective heat capacity c^* and heat transfer κ^*.

Solving a direct problem. Let's apply the recursive difference scheme for the ratio between the two temperature values calculated in the current and previous steps. Unknown temperature $T^*(t)$, obtained from equation (5) is determined using the recurrence relation as follows:

$$T^*(t_i) = \frac{\bar{Q}(t_i)\Delta t_i - c^* \Delta T_{ei} + T^*(t_{i-1})}{c^* + \kappa^* \Delta t_i}, \qquad (6)$$

where $\Delta t_i = t_i - t_{i-1}$ — time step, which corresponds to a moment in time t_i, $\Delta T_{ei} = T_e(t_i) - T_e(t_{i-1})$. In the equation (6) $\bar{Q}(t_i)$ is the discrete representation of the gas consumption function, which depends on the used heating mode (on/off). The possibility of discontinuities in this function caused by switching on and off the gas leads to the need to apply modified difference methods [7] that use smoothing procedures. In the calculations, information on the minimum duration of gas consumption was used to select the best step of the recursive scheme:

$$\sum_{i=1}^{N_p} \frac{\left| T^*(t_i)_{\Delta t_i^j} - T^*(t_i)_{\Delta t_i^{j-1}} \right|}{\left| T^*(t_i)_{\Delta t_i^j} \right|} \le e, \qquad (7)$$

where $T^*(t_i)_{\Delta t_i^j}$ — is the temperature value at the moment of time t_i, calculated by time step Δt_i, $\Delta t_i \in [0.1, 0.25, 0.5, 1]$, corresponding to 10, 15, 30 and 60 minute sampling steps, $e > 0$ is some sufficiently small value, N_p is total duration of measurements. To calculate the values at moments of time not specified during the measurement, it is necessary to use the numerical interpolation mechanism.

Although data on energy consumption is known throughout the year, information on the exact amount of energy used only for the heating process is not available. To solve this problem, the following heuristic algorithm was applied: 1) during the non-heating period, the gas consumption function was integrated, and the corresponding average value was set as the fraction of energy used in the heating period; 2) the calculated value was subtracted from the data for the cold period, and the corresponding calculated data were prepared for use in numerical experiments.

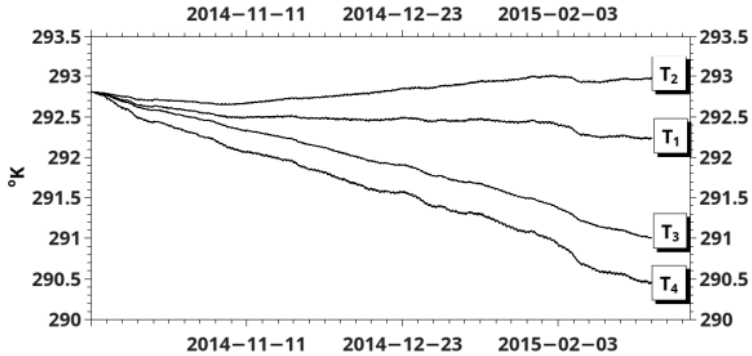

Fig. 2. Calculated temperature values for a set of parameters c^* and κ^*

Figure 2 shows the calculated temperature distributions in the room during the cold period (October 2014 - February 2015). These calculations were made for several pairs of parameters $\left\{c^*, \kappa^*\right\}$: $\left\{5 \cdot 10^3, 0.5\right\}$, $\left\{6 \cdot 10^3, 0.4\right\}$, $\left\{8 \cdot 10^3, 0.8\right\}$, $\left\{4 \cdot 10^3, 0.7\right\}$ corresponding to the temperatures T_1, T_2, T_3 and T_4. The input data for the calculations (initial indoor/outdoor temperature, energy consumption data) were selected from the REFIT data. Graphs in fig. 2 demonstrate significant sensitivity of the solutions to different parameter values c^* and κ^*.

In turn, in fig. 3 shows examples of solutions to equation (6), namely $T^*\left(t_i\right)$, calculated for different sets c^* and κ^*, given in the form of a grid of values: $c^* = \left[1.2 \cdot 10^3, 10^4\right]$, $\kappa^* = \left[10^1, 1\right]$. Here for the combination of parameters c^* and κ^* solutions $T^*\left(t_{100}\right)$ and $T^*\left(t_{500}\right)$ were calculated for moments of time t_{100} and t_{500} during the cold period. As one can see, each of the built surfaces falls to some minimum value, so to determine the optimal parameters c^* and κ^* it is possible to apply the minimization algorithm as part of solving the inverse problem.

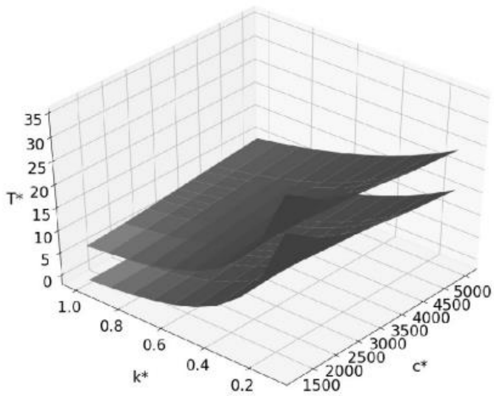

Fig. 3. Calculated temperature values $T^*\left(t_{100}\right)$ and $T^*\left(t_{500}\right)$ for a set of parameters c^* and κ^*

Solving and studying the inverse problem. Let's consider the approach to calculating effective thermal parameters c^* and κ^* based on solving the inverse problem formulated in the form of minimization of the following function:

$$\boldsymbol{\Phi}\left(c^*,\kappa^*\right)=\sum_{i=1}^{N_p}\left(\frac{T^*\left(t_i;c^*,\kappa^*\right)-T^*_{actual}\left(t_i\right)}{\left|T^*\left(t_i;c^*,\kappa^*\right)\right|}\right)^2 \to \min_{c^*,\kappa^*}, \tag{8}$$

where $T^*\left(t_i;c^*,\kappa^*\right)$ the calculated temperature as a solution of the corresponding direct problem (5) for the given parameters c^* and κ^*, $T^*_{actual}\left(t_i\right)$ is the real temperature value over time t_i. Here, the values of the difference between the calculated and real temperatures are normalized to prevent the accumulation of the squares of the values to an excessively large value due to the considerable length of the measurements.

This problem requires the application of a minimization algorithm to find the global minimum. For solving, we will use the brute force algorithm to obtain an initial approximation of the solution and the quasi-Newton method with limited memory BFGS [8] to obtain a more accurate solution.

To apply a rough search, we form a grid of parameter values c^* and κ^*: $c^*=\left[1.2\cdot10^3,10^4\right]$, $\kappa^*=\left[10^1,1\right]$ with some step for each parameter. We calculate the value for each pair from the grid of values $T^*\left(t_i;c^*,\kappa^*\right)$ taking into account the step according to (7) and determine the value of the functional $\boldsymbol{\Phi}$. The smallest of the obtained values on a given grid determines the initial approximation of the solution, which is used in the refinement procedure.

The refinement of the solution is implemented using the following quasi-Newton algorithm [8], which is based on the Broyden-Fletcher-Goldfarb-Shanno (BFGS) procedure. We reformulate problem (8) in the form

$$\boldsymbol{x}^* = \arg\min_{c^*,\kappa^*} \boldsymbol{\Phi}(\boldsymbol{x}), \tag{9}$$

$\boldsymbol{x} = \left(c^*, \kappa^*\right)$, which is solved on the basis of the following transformations

$$\boldsymbol{p}_i = -\boldsymbol{C}_i \nabla \boldsymbol{\Phi}(\boldsymbol{x}_i), \tag{10}$$

where $\boldsymbol{C}_i$ — matrix that represents an approximation to the inverse Hessian matrix in Newton's formula, $\nabla \boldsymbol{\Phi}(\boldsymbol{x}_i)$ — the gradient of the functional (8), the components of which can be approximately calculated by the finite difference method,

$$\boldsymbol{C}_{i+1} = \left(\boldsymbol{I} - \eta_i \Delta \boldsymbol{x}_i \boldsymbol{y}_i^T\right) \boldsymbol{C}_i \left(\boldsymbol{I} - \eta_i \boldsymbol{y}_i \Delta \boldsymbol{x}_i^T\right) + \eta \Delta \boldsymbol{x}_i \Delta \boldsymbol{x}_i^T, \tag{11}$$

where $\eta_i = \left(\boldsymbol{y}_i^T \Delta \boldsymbol{x}_i\right)^{-1}$, $\boldsymbol{I}$ — identity matrix, $\Delta \boldsymbol{x}_i = \boldsymbol{x}_{i+1} - \boldsymbol{x}_i$ — step of the method, $\boldsymbol{y}_i = \nabla \boldsymbol{\Phi}(\boldsymbol{x}_{i+1}) - \nabla \boldsymbol{\Phi}(\boldsymbol{x}_i)$. Each approximation of the solution (8) is calculated by the following formula, where $\alpha_i = \arg\min \boldsymbol{\Phi}(\boldsymbol{x}_i + \alpha \boldsymbol{p}_i)$, α — special parameter of the method, $\boldsymbol{x}_{i+1} = \boldsymbol{x}_i + \alpha_i \boldsymbol{p}_i$.

Since the solution of the inverse problem (8) is quite sensitive to the selected heating period, location of rooms and other features of the building, the range of calculated values of and can be quite wide.

Numerical experiments performed on the available data revealed that the value varies from 1200 to 4200 J/(m^{3}K), and the value from 0.6 to 0.65 W/(m^{3}K) for the periods December 2014 - February 2015 and November 2014 - February 2015 respectively.

Solving the inverse problem, c^* and κ^*, can be used to forecast energy consumption and formulate approaches to its reduction. Due to the sensitivity of such a solution to input data, it is necessary to pay attention to the improvement of preprocessing methods, for example, using the gas consumption disaggregation algorithm [9]. Also, the proposed approach makes it possible to approximate the effective thermophysical characteristics of architectural structures, based only on the measurement of temperatures and indicators of energy consumption of space heating.

Conclusion. The work proposes a methodology and a corresponding mathematical apparatus for an approximate assessment of the effective thermophysical parameters of the building: coefficients of thermal conductivity and heat capacity. For this purpose, a mathematical model and the corresponding direct heat conduction problem were formulated, the solution of which is based on the finite difference method with adaptive step selection. During numerical experiments, sufficient sensitivity of the solution (calculated room temperatures) to the effective thermophysical parameters was revealed. This makes it possible to formulate an inverse problem for estimating these parameters based on solving a number of direct problems. A numerical algorithm for the inverse problem is built, which involves a combination of the brute force approach to obtain approximate results and the BFGS algorithm based on the quasi-Newton method for their further refinement. The given results demonstrate the promising approach of the proposed approach to determining the effective thermophysical parameters of buildings. These results and the corresponding discussion are based on the materials of the dissertation [5].

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МЕТОДИКА ОЦІНКИ ТЕПЛОФІЗИЧНИХ ХАРАКТЕРИСТИК БУДІВЕЛЬ НА ОСНОВІ ОБЕРНЕНОЇ ЗАДАЧІ ТЕПЛОПРОВІДНОСТІ

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Як правило, під час проектування систем клімат-контролю для будівель вимірювання (такі як температура, вологість, споживання енергії тощо) використовуються та аналізуються за допомогою статистичних методів. Згодом ці дані можна використовувати для моделювання та створення інтелектуальних систем. Незважаючи на ефективність цих систем, часто буває вигідно включати фізичні рівняння в розроблені моделі. Ці рівняння не тільки описують теплову поведінку всередині приміщень, але також можуть бути інформативно інтегровані з алгоритмами аналізу даних.

Наприклад, знання характеристик стін, таких як їхня теплоємність і провідність, дозволяє точно регулювати параметри опалення в окремих приміщеннях і використовувати ці дані для розробки інтелектуальних термостатів. Ці характеристики можна визначити шляхом вирішення відповідних рівнянь теплопровідності. Однак, оскільки рівняння теплопровідності містить невідомі параметри (або параметри зі значеннями, які важко визначити), такі як коефіцієнти теплопровідності та теплоємності, існує потреба сформулювати обернені задачі для оцінки цих невідомих характеристик.

У роботі викладено методику розрахунку ефективних теплофізичних параметрів будівлі шляхом аналізу коливань зовнішньої та внутрішньої температури в холодну пору року, а також даних про споживання енергії на опалення. Метод скінченних різниць використовується тут для вирішення прямої проблеми, яка передбачає визначення теплового режиму в будівлі з урахуванням відповідних теплофізичних властивостей матеріалів. У роботі також представлена чисельна схема розрахунку етапів методу та досліджено вплив теплофізичних параметрів на розрахункові значення температури.

За результатами прямої задачі сформульовано та розв’язано обернену задачу ідентифікації ефективних теплотехнічних параметрів будівлі. Щоб вирішити цю проблему, для отримання наближеного рішення використовуються методи грубого перебору, а для його подальшого вдосконалення використовується квазіньютонівський алгоритм BFGS.

Ключові слова: теплофізичне моделювання, прямі та обернені задачі теплопровідності, чисельна оптимізація.

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