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SPECTRUM TRANSFORMATION OF THE RESTORED SIGNAL WITH REGULAR AND IRREGULAR SAMPLING

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The paper deals with the spectrum transformation of the signal restored by regular and irregular sampling. Irregular sampling is studied as a method of obtaining the noise-like error spectrum of the error of a restored signal. Hence the order of the output low-pass filter can be reduced, or in certain cases, this filter can be omitted. The most interesting area of application for this method may be the reproduction of a digital bitmap image. To simplify the problem, the error spectrum transformation is studied for the one-dimension sampling case.

Keywords: randomization, oversampling, spectrum, low-pass filter, LPF, noise, Nyquist criterion.

Introduction. The usual method for restoring a continuous signal from the sequence of discrete samples is the use of the low-pass filter (LPF), which significantly suppresses frequencies higher than half the sampling rate (Nyquist criterion). Ideally, such filter should be based on the function sinc(x) = sin(x)/x. However, real filters may use an approximation to this function, implemented in the digital region after the oversampling of the input signal, usually by an integer factor. In this case, a much simpler analog LPF can be installed after the output of the DAC. For the reproduction of bitmap images with a monitor or a printer, this final low-pass filtering indeed takes place during the direct observation of the image, thanks to the finite spatial resolution of the eye [1]. Thus, the period of the pixel lattice of the restored image should be sufficiently smaller than the limit of visual acuity at the particular observation distance.

For the usual observation conditions, this boundary may vary from 0,072 mm to 0,25-0,28 mm. An accepted resolution for printed bitmap images should be at least 300 dpi. The oversampling of the input image is required in all cases where the density of pixels in it is insufficient to fulfill this condition. Therefore, various interpolation algorithms are used. However, the interpolation of higher orders (for example, bicubic) or filters which approximating the function *sinc(x)* (for example, Lanczos) is not an universal method because of the possibility of creating visual artifacts such as extra contours that impair visual perception of the image and can complicate its analysis. Therefore, it is expedient to create an oversampling algorithm of signals or images in which the final error would be similar to random noise and did not differ from the additive noise that is already necessarily present.

Experimental model

The signal sampling model under studies consists of two stages.

At the first stage, the definition area is divided by a given number of intervals n. The number of intervals is determined based on [2] and was calculated by the formula $n = 2N$, where N is a positive integer number. In the second stage, each interval was divided into an odd number of samples m, which determines the instant value of the signal sample. The optimal number of samples was determined experimentally in order to provide a sufficient level of quality of the restored signal. Values of the sampled signal in each interval were determined as the arithmetic mean of all samples in this interval:

$$A_i = \frac{1}{m}\sum_{j=0}^{m} a_{ij}$$

The time parameter for this value was determined depending on the sampling mode:

- regular - the sample value corresponds to the central subsample in this interval.

- irregular - the sample value is placed randomly within the permissible deviation range. The limits of the deviation range are determined based on the randomization coefficient relative to the central sample of the interval. Thus, the random distribution, or jitter, is forcibly introduced into a regular distribution of interpolation nodes. This jitter is always present in the real processes of regular sampling of signals in the time area as result of the instability of the time scale of the clock generator and the position of the pulse fronts. It leads to the additional error of the discrete representation of the analog signal, so it is usually considered undesirable and the most signal processing systems minimize it [2]. On the one side, the additive noise that blurs the least significant bits of ADC resolution increases the quantization error and reduces the dynamic range of the data collection system. On the other side, this noise is a very powerful tool for converting nonlinear distortions related to the quantization error of low-level signals.

This two-stage model also allows one to determine the optimal parameters for restoring a signal with given accuracy more efficiently. Otherwise, this approach is closer to the physical principles of signal measurement, since any device adds some delay in the process of signal sampling.

Experimental results. The restoration of the signal by its discrete values was carried out using several types of interpolation: nearest-neighbor, linear, and cubic.

For the exploration, we chose a simple sinusoidal signal $y = \sin(x)$ (Fig. 1) because it meets the requirements, and also has well-known characteristics that ensure the authenticity of checking its recovery. In addition, we considered the case of restoring a real signal coming from the physical system and in which there is a random noise, which called the dither. This noise must be necessarily added to signal to be sampled to reduce distortion of the quantization of low-amplitude signals [3]. One period of the sinusoidal signal was sampled, being divided into n = 128 intervals, each of which consisted of 65 samples.

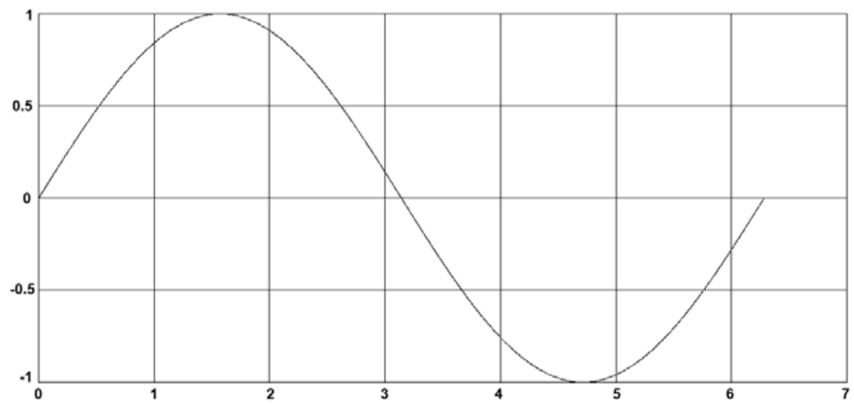

Fig. 1. The original signal $y = \sin(x)$.

The results of restoring the original signal with nearest neighbor, linear and cubic interpolations are shown on Fig. 2, 3, and 4, respectively. For restoring of the original signal with irregular sampling, we considered two cases with a randomization radius of 0,17 (Fig. 5-6) and 0,35 (Fig. 7-8). To modeling a real signal with random noise, the original signal $y = \sin(x)$ was used, which was distorted by a slight random noise.

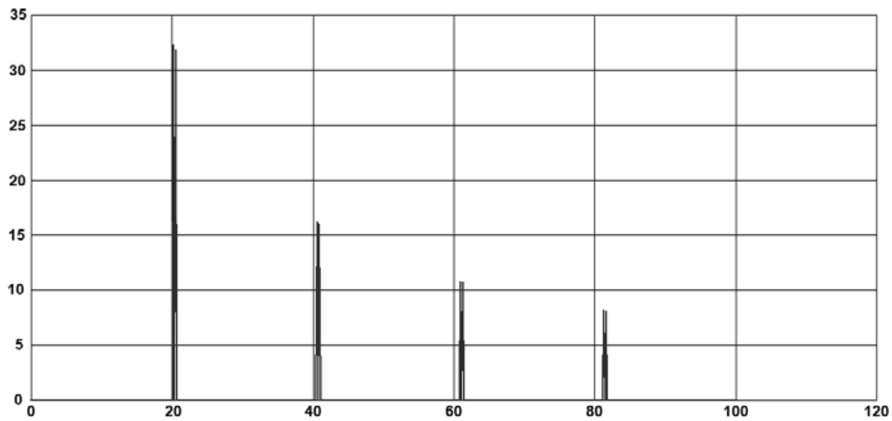

Fig. 2. Fragment of the Fourier spectrum of the nearest-neighbor interpolation of the original signal.

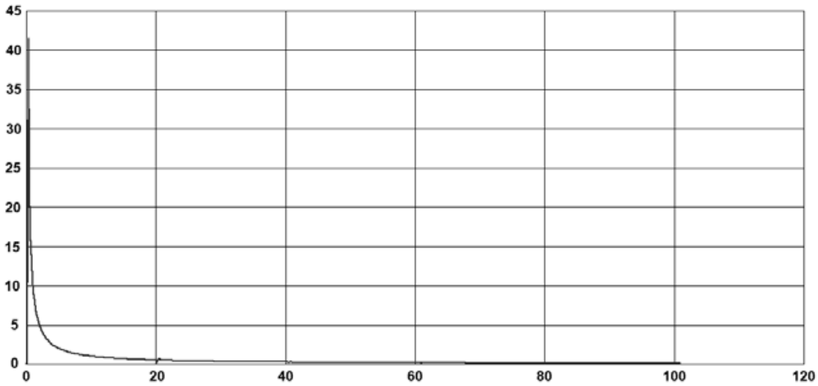

Fig. 3. Fragment of the Fourier spectrum of the linear interpolation of the original signal.

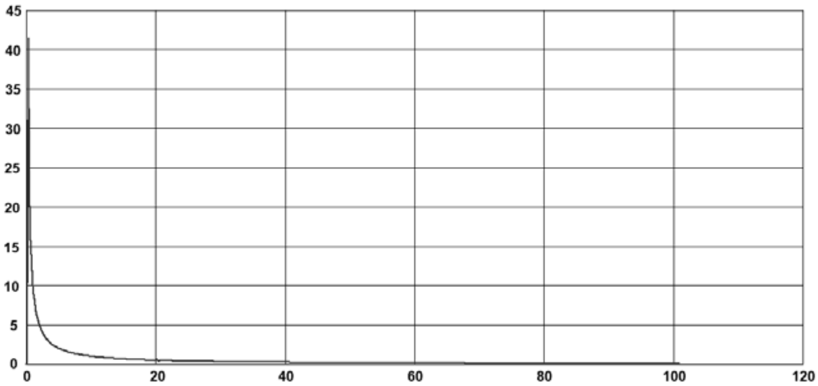

Fig. 4. Fragment of the Fourier spectrum of the cubic interpolation of the original signal.

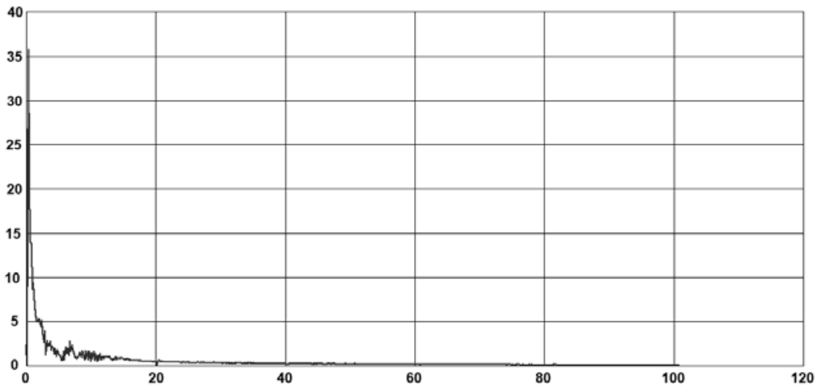

Fig. 5. Fragment of the Fourier spectrum of linear interpolation of a randomized signal with a randomization radius of 0,17.

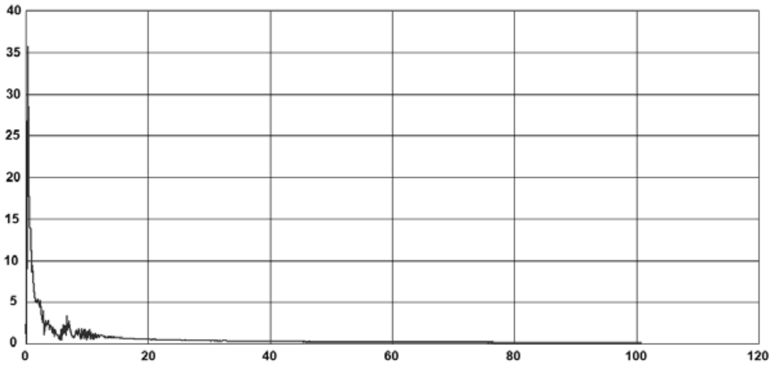

Fig. 6. Fragment of the Fourier spectrum of cubic interpolation of a randomized signal with a randomization radius of 0,17.

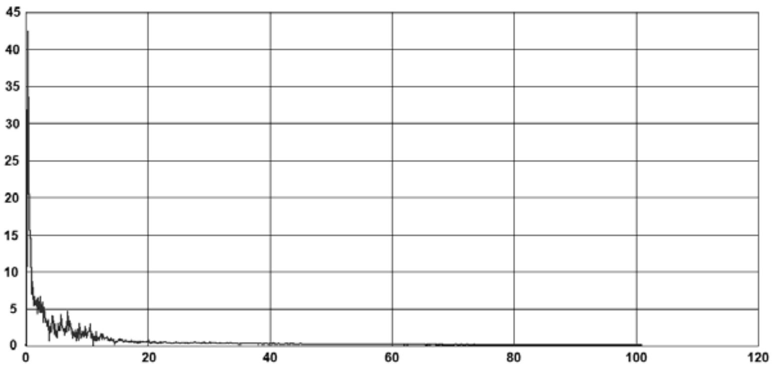

Fig. 7. Fragment of the Fourier spectrum of linear interpolation of a randomized signal with a randomization radius of 0,35.

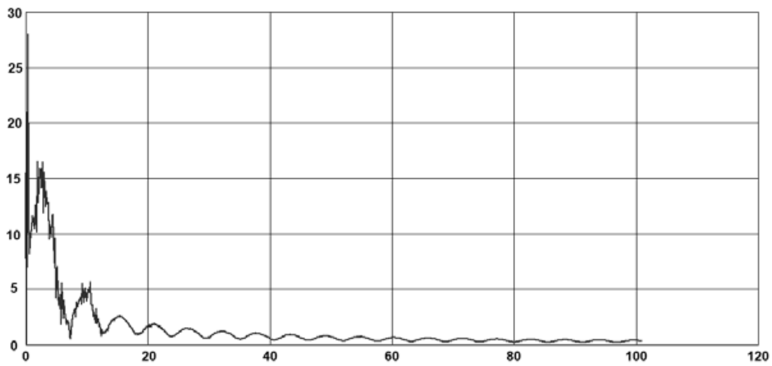

Fig. 8. Fragment of the Fourier spectrum of cubic interpolation of a randomized signal with a randomization radius of 0,35.

For the numerical evaluation of the results of the signal restoration, the squared difference between the original signal and the restored was calculated. Based on the received data, the total restoration error (see Table 1) and the maximum deviation of the restored signal (see Table 2) were calculated.

The results obtained reveal an increase of the error of restoring the original signal in the cases of randomization usage and also allow estimating the influence of the randomization radius on increasing the restoration error. Nevertheless, the relative changes in the restoration error indicate that the methods for signal restoration with randomization are less sensitive to the random noise what is present in the original signal. Furthermore, slight change of the maximum deviation of the restored signal (see Table 2) allows asserting that, even if the restoration error is increasing in the case of randomization usage, this error is random and largely blurs in the all domain of the restored signal. With the regular interpolation, we have an inverse case: the restoration error distributes only in the certain areas which are different for each interpolation method. The only exception was the maximum deviation of the signal restored using cubic interpolation with a randomization radius of 0.35, but this case can be explained by oscillations which appeared at the restored signal and are clearly shown on the Fourier spectrum (see Fig. 8).

Table 1.

Restoration error of the original signal.

Method of the signal restoration	The presence of random noise in the original signal		Relative change of the restoration error
	Without noise	With noise	
Regular nearest-neighbor interpolation	0,82618	0,841126	1,018
Regular linear interpolation	0,005711	0,015391	2,695
Regular cubic interpolation	0,004349	0,015596	3,586
Linear interpolation with a randomization radius of 0,17	0,022387	0,032527	1,453
Cubic interpolation with a randomization radius of 0,17	0,025457	0,03611	1,419
Linear interpolation with a randomization radius of 0,35	0,057032	0,087196	1,529
Cubic interpolation with a randomization radius of 0,35	0,072392	0,111122	1,535
Linear interpolation with a randomization radius of 0,50	0,126756	0,137172	1,082
Cubic interpolation with a randomization radius of 0,50	0,177568	0,20043	1,129

Table 2.

The maximum deviation of the restored signal.

Method of the signal restoration	The presence of random noise in the original signal	
	Without noise	With noise
Regular nearest-neighbor interpolation	0,000625	0,000729
Regular linear interpolation	0,000004	0,000016
Regular cubic interpolation	0,000001	0,000016
Linear interpolation with a randomization radius of 0,17	0,000025	0,000025
Cubic interpolation with a randomization radius of 0,17	0,000036	0,000036
Linear interpolation with a randomization radius of 0,35	0,000064	0,000081
Cubic interpolation with a randomization radius of 0,35	0,000064	0,0001
Linear interpolation with a randomization radius of 0,50	0,000144	0,000169
Cubic interpolation with a randomization radius of 0,50	0,000169	0,000256

Conclusion. The results reveal the conversion of the reproduction error depending on the selected parameters of the sampling. The nature of the error has changed and become more noise-like after the randomization. In the case of real signals with random noise, the randomization resulted in redistributing the error and decreasing the effects of random noise. However, randomization requires fine tuning, as the larger radius of randomization can create additional defects and increase the error significantly.

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ПЕРЕТВОРЕННЯ СПЕКТРА ВІДНОВЛЕНОГО СИГНАЛУ З РЕГУЛЯРНОЮ ТА НЕРЕГУЛЯРНОЮ ДИСКРЕТИЗАЦІЄЮ

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Описано характер перетворення спектра відновленого сигналу внаслідок регулярної і нерегулярної дискретизації. Як метод відновлення сигналу розглянуто три інтерполяційні методи різного порядку: східчасте інтерполювання, або інтерполювання методом найближчого сусіда, лінійна інтерполяція та кубічна інтерполяція. Використання різних методів відновлення дало змогу не лише перевірити якість відновлення сигналу, а й дослідити вплив рандомізації на методи відновлення, а також на появу різноманітних дефектів, таких як ефект дзвона чи осциляції. Нерегулярну дискретизацію розглянуто як метод трансформації спектра похибки відновленого сигналу з когерентного в шумовий, що дає змогу зменшити порядок вихідного фільтра низьких частот або, у деяких випадках, позбутися його сумісно з наддискретизацією сигналу, що відновлюється. Найцікавішим прикладом застосування таких методів може бути відтворення цифрового растрового зображення. Однак задля спрощення проблеми розглянуто одновимірний випадок, а для відновлення взято простий синусоїдальний сигнал, що дає змогу точно оцінити результати відновлення. Розглянуто випадок відновлення ідеального досліджуваного сигналу без шумів і спотворень, а також відновлення досліджуваного сигналу, у якому наявний випадковий шум – дітер. Такий шум характерний для більшості фізичних систем і може значною мірою впливати на якість відновлення сигналу. На підставі отриманих даних виконано обчислення похибки відновлення сигналу та максимального відхилення відновленого сигналу, які дають змогу оцінити точність відновлення сигналу в разі використання регулярної і нерегулярної дискретизації та демонструють трансформацію когерентної похибки відновлення сигналу у шумоподібну. Також продемонстровано зменшення негативного впливу випадкового шуму, що може бути наявний у вхідному сигналі, на процес відновлення сигналу.

Ключові слова: рандомізація, наддискретизація, спектр, фільтр низьких частот, шум, критерій Найквіста.