

УДК 539.3:338.45:622.32

## PLACEMENT OF WELLS AS A METHOD OF OIL FIELD DEVELOPMENT CONTROL

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Maintaining a profitable production on the depleted fields with high water cut production is one of the challenges encountered in the oil industry. The main objective of well placement optimization which provide a minimizing the economic costs is the recognition of the technological state of the object state under limited information. The present article is concerned with initial boundary value problem for nonlinear parabolic equations using the Leray-Schauder theorem and shows the existence and uniqueness of solutions of a finite mathematical programming problem with constraints of a special kind equality and inequalities.

*Key words:* oil and gaswell, well placement, oil reservoir, optimality criterion, uniqueness.

**1. Introduction.** Maintaining profitable production on the mature oil fields with high water production is one of the contemporary challenges faced by the oil industry. Ensuring adequate investments return by using a traditional (heuristic) methods of production control is a difficult task.

One of the most effective ways that can improve the oil production and provide the development of weak drainable oil reservoirs is infill development deposits by drilling new wells. Such activities include decision-making elements in order to ensure the justification of the information to find the optimal solution of optimizing new wells placement.

This paper is focusing on maximizing oil revenues during reservoir flooding, optimizing the medium and long term management of well placement, and operating wells.

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2010 Mathematics Subject Classification: 35K20, 76N25

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This work performed under the Research Program of the Azerbaijan National Academy of Sciences on “Complex theoretical and experimental studies of interdisciplinary problems of geomechanics” approved by Decree of the Presidium of the Azerbaijan National Academy of Sciences of №5/3, February 11, 2015 (2015 - 2017).

Within the framework of existing procedures, determination of the wells placement, its control is usually performed sequentially. In some cases, the strategy of reactive control, which depends on oil prices and the cost of the water production is implemented. This strategy entails the shutting-in wells for economic threshold criterion, but cannot provide the satisfactory solution since it does not take into account the relationship between the wells placement and related controls.

The paper proposes a joint approach to optimize well placement and conditions of their control. Two different optimizations in embedded mode are the well placement optimization repeatedly which alternates with optimization of well control.

The suite of methods used in optimization problems allows to specify a reservoir heterogeneity pattern, both geologically and in the hydrodynamic aspects, to identify areas where wells are competing for the same volume of fluid.

**2. Problem Statement.** To describe the two-dimensional flow weakly compressible oil in porous media was formulated as boundary value problem [1, 2]:

$$c(x) \frac{\partial p}{\partial t} - \operatorname{div}(a(x, \nabla p) \nabla p) + \sum_{l=1}^L q^l(t) \delta(x - x^l) = 0, \quad (1)$$

$$x \in \Omega^0 \subset E^2, \quad t \in (0, T],$$

$$p(x, 0) = p_0(x), \quad x \in \Omega;$$

$$p(x, t)|_{x \in \Gamma_1} = p_1(x, t), \quad \left. \frac{\partial p(x, t)}{\partial n} \right|_{x \in \Gamma_2} = p_2(x, t), \quad t \in (0, T], \quad (2)$$

$$a(x) = \frac{k(x)h(x)}{\mu}, \quad c(x) = h(m_0\beta_j + \beta_n), \quad (3)$$

where  $p = p(x, t)$  is the pressure at the point  $x \in \Omega$  at time  $t$ ;  $\Omega$  the flow area with boundary  $\Gamma$ , consisting of disjoint parts  $\Gamma_1, \Gamma_2$ ;  $\Gamma = \Gamma_1 \cup \Gamma_2, \Gamma_1 \cap \Gamma_2 = \emptyset$ ;  $\Omega^0 = \Omega \setminus \Gamma$ ;  $k(x)$  the permeability;  $h(x)$  the bed thickness;  $\mu$  the fluid viscosity;  $m$  the porosity;  $\beta_j, \beta_n$  the coefficients of fluid and porous medium compressibility;  $x^l = (x_1^l, x_2^l)$  the coordinates of  $l$ -th well placements with a flow rate  $q^l(t), l = 1, 2, \dots, L$ ,  $L$  the number of wells;  $\delta(\cdot)$  the the generalized two-dimensional Dirac delta function;  $T$  the planning period.

**3. Problem-solving procedure.** It is assumed that all functions, reservoir, and the fluid parameters involved in the initial-boundary value problem (1)–(3), including the coordinates of the wells  $x^l, l = 1, \dots, L_1$ , are defined. Wells  $L_2$  with unknown coordinates  $x_i$ , where  $i = L_1 + 1, \dots, L_1 + L_2 = L$ , is necessary to put into exploration, keeping the following performance, geological and scheduled targets as:

$$p(x, t)|_{x \in \Gamma_1} = p_1(x, t), \quad \left. \frac{\partial p(x, t)}{\partial n} \right|_{x \in \Gamma_2} = p_2(x, t), \quad t \in (0, T], \quad (4)$$

$$(x_1^i, x_2^i) \in \Omega, \quad i = L_1 + 1, \dots, L; \quad (5)$$

$$\|x^i - x^j\| \geq D, \quad i, j = 1, \dots, L, \quad i \neq j; \quad (6)$$

$$0 \leq q^l \leq q^l(t) \leq q^{-l}, \quad l = 1, \dots, L; \quad (7)$$

$$\sum_{l=1}^L \int_0^T q^l(t) dt \geq q, \quad (8)$$

where  $\|\cdot\|$  is the Euclidean norm on the plane;  $D$  the minimum distance between wells;  $q_p$  the target for oil production. As can be seen from (5)–(7), in the performance of a problem with new wells placement, operational conditions must be considered and also optimized. Although some of the working flow rates of wells are set, it is not recommended to change them.

As a condition let us write

$$a(x, \zeta)\zeta > C_1|\zeta|^2; \quad C_1 = \text{const}, \quad \zeta \in E^2. \quad (9)$$

Equations (5)–(8) show that in addition to the problem of new wells placement, it is also taken into account the possibility of wells optimization. It should also be noted that the performance of some wells is pre-defined and there is no need to assign them new flow rates performance.

As optimum, the reservoir pressure, minimizing changes in the porous medium permeability, reservoir fluid viscosity, and maximizing oil production criteria are accepted, including their combinations and multi-criteria cases.

**4. Conclusions.** Thus, in this study, the initial boundary value problem for a nonlinear parabolic equation was defined. This ensures that the imposed conditions and the boundary conditions are mixed, i.e. on the one side is the Dirichlet conditions, on the other the Neumann conditions. Under conditions (5)–(9) with the aid of Leray-Schauder theorem [6, 7], the existence and uniqueness of (1)–(3) problem solution was proven.

The theoretical analysis, presented in this paper, provides minimization of the deviations from the average residual reservoir energy. The appropriate functional finite-difference approximation of the whole problem is performed. Thus, the goal – to obtain a finite mathematical programming problem with constraints of special kind of equality – was achieved, and inequalities, which belongs to a class of optimization problems of network structure. We also use the combination of the method of exterior penalty functions for the account of constraints (4), (5) and the projection of conjugate gradient of the penalty functional taking into account accommodate linear and positional constraints (6) and (7) to solve the resulting problem of mathematical programming. The boundary value problem is approximated using the nets technique and the numerical results for this problem is presented.

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*Стаття: надійшла до редколегії 18.11.2015  
прийнята до друку 14.12.2016*

## ОПТИМАЛЬНОЕ РОЗТАШУВАННЯ СВЕРДЛОВИН ДЛЯ УПРАВЛІННЯ ПРОДУКТИВНІСТЮ НАФТОВОГО ПОЛЯ

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Найважливіша прблема нафтовидобутку – забезпечити економічно вигідну експлуатацію вироблених нафтових полів в умовах обмеженої поінформованості. Відповідна проблема формулюється як двовимірна мішана крайова задача механіки пористих середовищ, у якій параметрами є пористість нафтомісткого середовища, в'язкість і тиск нафти. Оптимізується розташування свердловин і параметри контролю за їхньою експлуатацією. Для застосованих оптимізаційних процедур із обмеженнями з використанням теореми Лере-Шаудера доведено існування та єдиність розв'язку.

*Ключові слова:* нафтові та газові свердловини, оптимальне розташування свердловин, нафтові родовища, оптимальні критерії, єдиність.