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EXTENSION OF FUZZY METRICS: ZERO-DIMENSIONAL CASE

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The main result of this note is a construction of an operator that extends fuzzy metrics defined on the closed sets of zero-dimensional fuzzy metrizable space over the whole space. The extension operator preserves the operation of minimum of fuzzy metrics as well as the operation of truncation. We also define a fuzzy metric on the countable product of fuzzy metric spaces.

Key words: fuzzy metric, extension operator, zero-dimensional space.

1. Introduction. The notion of fuzzy metric space is tightly related with that of probabilistic metric space introduced by Menger [6] (see also [7]). The fuzzy metric spaces were defined in the paper [5] and later their definition was modified in [8]. The version from [8] is more restrictive. However, it turns out that the fuzzy metrics in the sense of [8] determine the metrizable topologies.

It is well-known that the family of metrics on a set forms a cone with respect to the operations of sum and product with the non-negative scalar. Also, the maximum of two metrics is a metric. We establish counterparts of these properties for the fuzzy metrics.

One of the main results of this note is the construction that allows, in the zero-dimensional case, to extend fuzzy metrics from a closed subset to the whole set. Remark that, in the case of metrics, the problem of extension has a long history, which traces back to Hausdorff. The problem of existence of operators extending the cones of metrics was first formulated (and partially solved) by Bessaga [3]. Its complete solution is obtained by Banakh [1]; a short proof of existence can be found in [11].

It turns out that this extension operator preserves the mentioned operations on the fuzzy metrics. Our construction is based on fuzzy metrization of the countable product of the fuzzy metric spaces. We also prove that, for any two fuzzy metric spaces, there exists a fuzzy metrization of the bouquet of these spaces that agrees with these fuzzy metrics.

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2. Preliminaries. We start with some necessary definitions concerning the notion of fuzzy metric space.

A continuous t-norm is a continuous map $(x, y) \mapsto x * y \colon [0,1] \times [0,1] \to [0,1]$ which satisfies the following properties:

(1) $(x * y) * z = x * (y * z)$;
(2) $x * y = y * x$;
(3) $x * 1 = x$;
(4) if $x \le x'$ and $y \le y'$, then $x * y \le x' * y'$.

In other words, a continuous t-norm is a continuous Abelian monoid with unit 1 and with the monotonic operation. The following are examples of continuous t-norms:

(1) $x * y = \min\{x, y\}$;
(2) $x * y = \max\{0, x + y - 1\}$.

Definition 1. *A* fuzzy metric space *is a triple* $(X, M, *)$*, where* X *is a nonempty set,* $*$ *is a continuous t-norm and* M *is a fuzzy set of* $X \times X \times (0, \infty)$ *(i.e.* M *is a map from* $X \times X \times (0,\infty)$ *to* $[0,1]$*) satisfying the following properties:*

(i) $M(x, y, t) > 0$*;*
(ii) $M(x, y, t) = 1$ *if and only if* $x = y$*;*
(iii) $M(x, y, t) = M(y, x, t)$*;*
(iv) $M(x, y, s) * M(y, z, t) \le M(x, z, s + t)$*;*
(v) *the function* $M(x, y, -) \colon (0, \infty) \to (0, 1]$ *is continuous.*

We obtain the notion of a fuzzy pseudometric space if we replace condition (ii) from the above definition by the following condition:

(ii') $M(x, x, t) = 1$.

In a fuzzy metric space $(X, M, *)$, we say that the set

$$B_M(x, r, t) = \{y \in X \mid M(x, y, t) > 1 - r\}, \quad x \in X, \quad r \in (0,1), \quad t \in (0, \infty),$$

is the *open ball* of radius $r > 0$ centered at x for t. It is proved in [8] that the family of all open balls is a base of a topology on X; this topology is denoted by τ_M.

Proposition 1. *Let* $(X, M_i, *)$*,* $i = 1, 2$*, be fuzzy metric spaces. Then* $(X, M, *)$*, where* $M(x, y, t) = M_1(X, y, t) * M_2(x, y, t)$*, is also a fuzzy metric space.*

Proof. We are going to verify properties (i)–(iv) from Definition 1.

(i) Obvious.

(ii) Clearly, $M(x, x, t) = 1$, for every $x \in X$ and $t \in (0, \infty)$. If $M(x, y, t) = 1$, then

$$1 = M(x, y, t) = M_1(x, y, t) * M_2(x, y, t) \le M_1(x, y, t) * 1 = M_1(x, y, t),$$

whence $M_1(x, y, t) = 1$ and therefore $x = y$.

(iii) Obvious.

(iv) We have

$$\begin{aligned} M(x, y, s) * M(y, z, t) &= M_1(x, y, s) * M_2(x, y, s) * M_1(y, z, t) * M_2(y, z, t) = \\ &= M_1(x, y, s) * M_1(y, z, t) * M_2(x, y, s) * M_2(y, z, t) \le \\ &\le M_1(x, z, s + t) * M_2(x, z, s + t) = M(x, z, s + t). \end{aligned}$$

(v) Obvious.

Proposition 2. *Let $(X, M_i, *)$, $i = 1, 2$, be fuzzy metric spaces. Then $(X, M, *)$, where $M(x, y, t) = \min\{M_1(X, y, t), M_2(x, y, t)\}$, is also a fuzzy metric space.*

Proof. We are going to verify properties (i)–(iv) from Definition 1.

(i) Obvious.

(ii) Clearly, $M(x, x, t) = 1$, for every $x \in X$ and $t \in (0, \infty)$. If $M(x, y, t) = 1$, then $M_1(x, y, t) = M_2(x, y, t) = 1$ and therefore $x = y$.

(iii) Obvious.

(iv) We have

$$M(x, y, s) * M(y, z, t) = \min\{M_1(x, y, s), M_2(x, y, s)\} * \min\{M_1(y, z, t), M_2(y, z, t)\} \leq$$
$$\leq M_i(x, y, s) * M_i(y, z, t) \leq M_i(x, z, s + t), \quad i = 1, 2,$$

whence $M(x, y, s) * M(y, z, t) \leq M(x, z, s + t)$.

(v) Obvious.

One can similarly prove the following statement.

Proposition 3. *Let $(X, M_\alpha, *)$, $\alpha \in \mathcal{A}$, be fuzzy metric spaces. Suppose that, for every $x, y \in X$, $x \neq y$, we have $\inf\{M_\alpha(x, y, t) \mid \alpha \in \mathcal{A}\} < 1$. Then $(X, M, *)$, where*

$$M(x, y, t) = \inf\{M_\alpha(x, y, t) \mid \alpha \in \mathcal{A}\},$$

is also a fuzzy metric space.

Proposition 4. *Let $(X, M, *)$ be a fuzzy metric space and $c \in (0, 1)$. Then $(X, M', *)$, where $M'(x, y, t) = \max\{M(x, y, t), c\}$, is also a fuzzy metric space.*

Proof. The only condition from Definition 1 which requires verification is (iv). We are going to prove that

$$M'(x, y, s) * M'(y, z, t) \leq M'(x, z, s + t). \tag{1}$$

The proof splits into three cases.

a) $M(x, y, s) \leq c$. Then (1) reduces to the following:

$$c * M'(y, z, t) \leq M'(x, y, s + t).$$

Since

$$c * M'(y, z, t) \leq c * 1 \leq c \leq M'(x, z, s + t),$$

we are done.

b) $M(x, y, s) > c$, $M(y, z, t) > c$. Then

$$M'(x, y, s) * M'(y, z, t) = M(x, y, s) * M(y, z, t) \leq M(x, z, s + t) \leq M'(x, z, s + t).$$

c) $M(x, y, s) > c$, $M(y, z, t) \leq c$. Then

$$M'(x, y, s) * M'(y, z, t) \leq M(x, y, s) * c \leq c \leq M'(x, z, s + t).$$

In the sequel, we use the notation $c \odot M$ for the fuzzy metric $\max\{M(x, y, t), c\}$.

Remark 1. Counterparts of Propositions 1-4 are also valid for the fuzzy pseudometric spaces.

2.1. Fuzzy metrics on bouquets. Let $X = X_1 \vee X_2$ and let a be the base point of X. Let M_i be fuzzy metrics on X_i, $i = 1, 2$, (with respect to the same t-norm $*$). Define the symmetric with respect to the first and the second variable function $M\colon X \times X \times (0, \infty) \to \to [0, 1]$ as follows:

$$M(x, y, t) = \begin{cases} M_i(x, y, t), & \text{if } x, y \in X_i,\ i = 1, 2, \\ \sup\{M_1(x, a, t_1) * M_2(a, y, t_2) \mid t_1 + t_2 = t\}, & \text{if } x \in X_1, y \in X_2. \end{cases}$$

Proposition 5. *The function M is a fuzzy metric on X with respect to the t-norm $*$. The topology induced by M is that of the bouquet topology on $X = X_1 \vee X_2$.*

Proof. Clearly, $M(x, x, t) = 1$, for every $x \in X$. Suppose now that $M(x_1, x_2, t) = 1$ and $x_1 \neq x_2$. Then, without loss the generality, one may assume that $x_i \in X_i \setminus \{a\}$, $i = 1, 2$. Then $M_1(x_i, a, t) < 1$, $i = 1, 2$, whence

$$\begin{aligned} M(x_1, x_2, t) \leq &\sup\{M_1(x, a, t_1) * M_2(a, y, t_2) \mid t_1 + t_2 = t\} \leq \\ \leq &M_1(x_1, a, t) * M_2(a, x_2, t) < 1 * 1 = 1 \end{aligned}$$

and we obtain a contradiction.

(iii) We have to prove that, for all $x, y, z \in X$ and $t, s \in (0, \infty)$,

$$M(x, y, t) * M(y, z, t) \leq M(x, z, t + s).$$

We consider two cases. 1) $x, y \in X_1$, $z \in X_2$, then

$$\begin{aligned} M(x, y, t) * M(y, z, s) \leq &M_1(x, y, t) * (\sup\{M_1(y, a, s_1) * M(a, z, s_2) \mid s_1 + s_2 = s\}) = \\ = &\sup\{M_1(x, y, t) * (\{M_1(y, a, s_1) * M(a, z, s_2) \mid s_1 + s_2 = s\} \leq \\ \leq &\sup\{M_1(x, a, t + s_1) * M(a, z, s_2) \mid s_1 + s_2 = s\} = \\ = &\sup\{M_1(x, a, \tau_1) * M(a, z, \tau_2) \mid \tau_1 + \tau_2 = t + s,\ \tau_1 \geq t\} \leq \\ \leq &\sup\{M_1(x, a, \tau_1) * M(a, z, \tau_2) \mid \tau_1 + \tau_2 = t + s\} = \\ = &M(x, z, t + s). \end{aligned}$$

2) $x, z \in X_1$, $y \in X_2$. Then

$$\begin{aligned} M(x, y, t) * M(y, z, s) \leq &(\sup\{M_1(x, a, t_1) * M(a, y, t_2) \mid t_1 + t_2 = t\}) * \\ &* (\sup\{M_1(y, a, s_1) * M_2(a, z, s_2) \mid s_1 + s_2 = s\}) \leq \\ \leq &\sup\{M_1(x, a, t_1) \mid t_1 \leq t\} * \sup\{M_1(a, z, s_1) \mid s_1 \leq s\} \leq \\ \leq &M_1(x, z, t) = M(x, z, t). \end{aligned}$$

(iv) We are going to prove that, for any $x, y \in X$, the map $\gamma\colon t \mapsto M(x, y, t)$ is continuous. We only need to consider the case $x \in X_1$, $y \in X_2$. First, since the maps $t \mapsto M_1(x, a, t)$ and $t \mapsto M_2(a, y, t)$ are continuous and nondecreasing, there exist unique continuous extensions of these maps onto the set $[0, \infty)$. We preserve the same notations for the extended maps. Let us denote by $\varphi\colon [0, \infty) \times [0, \infty) \to [0, 1]$ the function acting by the formula:

$$\varphi(t_1, t_2) = M_1(x, a, -) * M_2(a, y, -).$$

The map

$$\alpha\colon t \mapsto \{(t_1, t_2) \in [0, \infty) \times [0, \infty) \mid t_1 + t_2 = t\}$$

is a continuous map from $[0,\infty)$ to the space $2^{[0,\infty)\times[0,\infty)}$ of nonempty compact subsets in $[0,\infty)\times[0,\infty)$; the latter is endowed with the Hausdorff metric d_H:

$$d_H(A,B)=\inf\{r>0\mid A\subset O_r(B),\ B\subset O_r(A)\}$$

(here O_r stands for the r-neighborhood with respect to the euclidean metric on $[0,\infty)\times[0,\infty)$). Now the map γ is the composition

$$(0,\infty)\xrightarrow{\quad\alpha\quad}2^{[0,\infty)\times[0,\infty)}\xrightarrow{\ \sup\_\ \varphi\ }[0,1]\ ,$$

where the function $\sup\_\ \varphi$ assigns to every $A\in 2^{[0,\infty)\times[0,\infty)}$ the number $\sup\{\varphi(x)\mid x\in A\}$; the function $\sup\_\ \varphi$ is known to be continuous (see, e.g., [4]). Therefore, γ is continuous.

It is clear that the fuzzy metric M induces the bouquet topology on X.

Remark 2. The proof of Proposition 5 can be immediately generalized over the case of bouquet of arbitrary number of fuzzy metric spaces.

3. Extension of metrics.

3.1. Fuzzy metrics on the countable powers. Let $(X_i,M_i,*)$, $i\in\mathbb{N}$, be a family of fuzzy metric spaces, $X=\prod_{i\in\mathbb{N}}X_i$.

Theorem 1. *The function $\bar M\colon X\times X\times(0,\infty)\to[0,1]$ defined by the formula*

$$\bar M((x_i),(y_i),t)=\inf\{(1/i)\odot M(x_i,y_i,t)\mid i\in\mathbb{N}\}$$

is a fuzzy metric on X. The topology τ_M coincides with the product topology on X generated by the fuzzy metrics τ_{M_i}, $i\in\mathbb{N}$.

Proof. Let us denote by $p_i\colon X\to X_i$ the projection onto the i-th coordinate, $i\in\mathbb{N}$. By Proposition 4, the function $M_i'\colon X_i\times X_i\times(0,\infty)\to[0,1]$ defined by the formula $M_i'(x,y,t)=M_i(p_i(x),p_i(y),t)$ is a fuzzy pseudometric on X_i. By the remark after Proposition 2, $\bar M$ is a fuzzy pseudometric on X.

Let $x,y\in X$, $x\neq y$, then there exists $i\in\mathbb{N}$ such that $p_i(x)\neq p_i(y)$. Therefore $M_i'(x,y,t)<1$ and consequently $\bar M(x,y,t)<1$, for every t. This shows that $\bar M$ is a fuzzy metric on X.

Let us use $\bar B$ to denote the balls with respect to the fuzzy metric $\bar M$ and B_i to denote the balls with respect to the fuzzy metric M_i.

Let $x,y\in X$, $r\in(0,1)$, and $t\in(0,\infty)$. Let $x_i=p_i(x)$, $y_i=p_i(y)$. If $y\in\bar B(x,r,t)$, then $\bar M(x,y,t)>1-r$ and therefore, there exists $\varepsilon\in(0,1-r)$ such that

$$\inf\{(1/i)\odot M_i(x_i,y_i,t)\mid i\in\mathbb{N}\}>1-r+\varepsilon.$$

Let

$$K=\bigcup_{\varepsilon\in(0,1-r)}\left(\prod_{(1/i)\le 1-r+\varepsilon}B_i(x_i,r-\varepsilon,t)\times\prod_{(1/i)>1-r+\varepsilon}X_i\right).$$

We conclude that $y\in K$. Since all the implications above are reversible, we see that $\bar B(x,r,t)=K$.

Show that

$$K=\prod_{(1/i)\le 1-r}B_i(x_i,r,t)\times\prod_{(1/i)>1-r}X_i.$$

If $y \in K$ and $1/i \leq 1 - r$, then $(1/i) \leq 1 - r + \varepsilon$ and therefore

$$y_i \in B_i(x_i, r - \varepsilon, t) \subset B_i(x, r, t),$$

whence

$$y \in \prod_{(1/i) \leq 1-r} B_i(x_i, r, t) \times \prod_{(1/i) > 1-r} X_i.$$

On the other hand, let

$$y \in \prod_{(1/i) \leq 1-r} B_i(x_i, r, t) \times \prod_{(1/i) > 1-r} X_i.$$

Then there exists $\varepsilon > 0$ such that $y_i \in B_i(x_i, r + \varepsilon, t)$ for all i with $(1/i) \leq 1 - r$ and therefore $y \in K$.

We have proven that the topology on X generated by the fuzzy metric $\bar{M}$ on X is contained in the product topology on X generated by the fuzzy metrics M_i.

On the other hand, let

$$\prod_{i \leq n} B_i(x_i, r_i, t_i) \times \prod_{i > n} X_i$$

be a basic neighborhood of $x \in X$. Since the functions $M_i(a, b, -)$ are nondecreasing, we see that

$$x \in \prod_{i \leq n} B_i(x_i, r, t) \times \prod_{i > n} X_i \subset \prod_{i \leq n} B_i(x_i, r_i, t_i) \times \prod_{i > n} X_i,$$

where

$$r = \max\{r_1, \dots, r_n\}, \; t = \min\{t_1, \dots, t_n\}.$$

Choose $r' \in (\max\{r, 1 - \frac{1}{n}\}, 1)$, then

$$x \in \bar{B}(x, r', t) = \prod_{(1/i) \leq 1-r'} B_i(x_i, r', t) \times \prod_{(1/i) > 1-r'} X_i \subset \prod_{i \leq n} B_i(x_i, r, t) \times \prod_{i > n} X_i$$

and this allows us to conclude that the topology on X generated by $\bar{M}$ coincides with the product topology of the topologies generated by the fuzzy metrics M_i, $i \in \mathbb{N}$.

3.2. Extension of fuzzy metrics. Given a metrizable space X, let us denote by $\mathcal{FPM}(X)$ (respectively $\mathcal{FM}(X)$) the set of all fuzzy pseudometrics (respectively fuzzy metrics) on X compatible with the topology of X.

Let A be a closed subset of X. An *extension operator* for fuzzy (pseudo)metrics is a map $u\colon \mathcal{FM}(A) \to \mathcal{FM}(X)$ (respectively $u\colon \mathcal{FPM}(A) \to \mathcal{FPM}(X)$) such that $u(M)|(A \times A \times (0, \infty)) = M$, for every $M \in \mathcal{FM}(A)$ (respectively $M \in \mathcal{FPM}(A)$)

Theorem 2. *Let A be a closed subspace of a zero-dimensional separable metrizable space X, $|A| \geq 2$. Then there exists a fuzzy metric extension operator u that satisfies the following properties:*

(1) $u(\min\{M_1, M_2\}) = \min\{u(M_1), u(M_2)\}$;
(2) $u(c \odot M) = c \odot u(M)$.

Proof. Let $a, b \in A$, $a \neq b$. Consider a countable base $\{U_i \colon i \in \mathbb{N}\}$ of $X \setminus A$ consisting of open and closed in X sets. Let also $\{V_i \colon i \in \mathbb{N}\}$ be a countable family of open and closed subsets in X which forms a base of topology at all the points of A.

Since A is a closed subset of a zero-dimensional metrizable space, there exists a continuous retraction $r \colon X \to A$ (see, e.g., [4]).

Define a countable family $\mathcal{R} = \{r_i \mid i \in \mathbb{N}\}$ of continuous retractions of X onto A as follows:

$$\begin{aligned} r_{4i-3}|(X \setminus U_i) =& r_{4i-2}|(X \setminus U_i) = r|(X \setminus U_i), \\ r_{4i-3}(U_i) =& a, \quad r_{4i-2}(U_i) = b, \\ r_{4i-1}|(X \setminus (r^{-1}(V_i) \setminus V_i)) =& r_{4i}|(X \setminus (r^{-1}(V_i) \setminus V_i)) = r|(X \setminus (r^{-1}(V_i) \setminus V_i)), \\ r_{4i-1}(r^{-1}(V_i) \setminus V_i) =& a, \quad r_{4i}(r^{-1}(V_i) \setminus V_i) = b. \end{aligned}$$

Clearly, $r = (r_i)_{i \in \mathbb{N}} \colon X \to X^{\mathbb{N}}$ is continuous and injective. That the map r is an embedding easily follows from the fact that the set $\{r_i \mid i \in \mathbb{N}\}$ separates the points and the closed sets in X.

Let $M \in \mathcal{FM}(A)$. Denote by $\bar{M}$ the fuzzy pseudometric on $A^{\mathbb{N}}$ defined by the formula:

$$\bar{M}((x_i), (y_i), t) = \inf\{(1/i) \odot M(x_i, y_i, t) \mid i \in \mathbb{N}\}.$$

Define $u(M) \colon X \times X \times (0, \infty) \to [0, 1]$ by the formula: $u(M)(x, y, t) = \bar{M}(r(x), r(y), t)$. Since the map r is injective, we see that $u(M)$ is a fuzzy pseudometric on X; clearly, $u(M)$ is a fuzzy metric on X whenever $M \in \mathcal{FM}(A)$.

Let $x, y \in A$ and $t \in (0, \infty)$, then

$$\begin{aligned} u(M)(x, y, t) =& \bar{M}(r(x), r(y), t) = \inf\{(1/i) \odot M(r_i(x), r_i(y), t) \mid i \in \mathbb{N}\} = \\ =& \inf\{(1/i) \odot M(x, y, t) \mid i \in \mathbb{N}\} = M(x, y, t), \end{aligned}$$

i.e., $u(M)$ is an extension of M.

Given $M_1, M_2 \in \mathcal{FM}(A)$, we have

$$u(\min\{M_1, M_2\})(x, y, t) = \overline{\min\{M_1, M_2\}}(r(x), r(y), t) =$$

If $c \in (0, 1)$, then

$$\begin{aligned} u(c \odot M)(x, y, t) =& \overline{c \odot M}(r(x), r(y), t) = \inf\{(1/i) \odot c \odot M(x, y, t) \mid i \in \mathbb{N}\} \\ =& c \odot \inf\{(1/i) \odot M(x, y, t) \mid i \in \mathbb{N}\} = c \odot u(M)(x, y, t), \end{aligned}$$

thus (2) holds.

4. Remarks and open questions. Similarly as in [9], [10], one can consider the problem of simultaneous extension of fuzzy (pseudo)metrics defined on the closed subsets of a metrizable space. For any metric space (Y, d), let $\mathrm{CL}(Y)$ the family of all nonempty closed subsets of Y. We consider the following Wijsman convergence in $\mathrm{CL}(Y)$: a sequence (A_i) converges to A if, for any $y \in Y$, the sequence $d(y, A_i)$ converges to $d(y, A)$.

Given a fuzzy metric M defined on a set $A \in \mathrm{CL}(X)$ (we express this by writing $\mathrm{dom}(M) = A$), for a metric space X, identify every $M \in \mathcal{FM}(A)$ with its graph

$$\begin{aligned} \Gamma_M =& \{(x, y, t, r) \in A \times A \times (0, \infty) \times [0, 1] \mid r = M(x, y, t)\} \in \\ \in& \mathrm{CL}(X \times X \times (0, \infty) \times [0, 1]). \end{aligned}$$

We endow the set $\mathcal{FM} = \bigcup\{\mathcal{FM}(A) \mid A \in \mathrm{CL}(X)\}$ with the topology generated by the Wijsman convergence of their graphs.

Question 1. Is their a simultaneous extension operator $u\colon \mathcal{FM} \to \mathcal{FM}(X)$ (i.e. u satisfying the property

$$u(M)|(\mathrm{dom}(M) \times \mathrm{dom}(M) \times (0, \infty)) = M,$$

for every $M \in \mathcal{FM}$) which is continuous in the topology of Wijsman convergence?

A similar question can be formulated for the fuzzy pseudometrics.

One can consider also another topologies on the sets of closed subsets: Attouch-Wets, Hausdorff etc (see [2]).

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ПРОДОВЖЕННЯ РОЗМИТИХ МЕТРИК: НУЛЬВИМІРНИЙ ВИПАДОК

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Наша мета – побудова оператора, що продовжує розмиті метрики, означені на замкнених підпросторах нульвимірного розмитого метричного простору, на весь простір. Оператор продовження зберігає операцію мінімуму розмитих метрик, а також операцію обтинання. Подано також означення розмитої метрики на зліченному добутку розмитих метричних просторів.

Ключові слова: розмита метрика, оператор продовження, нульвимірний простір.

ПРОДОЛЖЕНИЕ НЕЧЕТКИХ МЕТРИК: НУЛЬМЕРНЫЙ СЛУЧАЙ

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Наша цель – построение оператора, продолжающего нечеткие метрики, определенные на замкнутых подпространствах нульмерного нечеткого метрического пространства, на все пространство. Оператор продолжения сохраняет операцию минимума нечетких метрик, а также операцию отсечения. Дано также определение нечеткой метрики на счетном произведении нечетких метрических пространств.

Ключевые слова: нечеткая метрика, оператор продолжения, нульмерное пространство.

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