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ASYMPTOTIC BEHAVIOR OF JOINT DISTRIBUTION OF THE OVERSHOOT AND THE FIRST PASSAGE TIME OF MARKOV CHAIN FOR THE LEVEL

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In the paper we prove a theorem on asymptotic behavior of joint distribution of overshoot and the first passage time of a Markov chain homogeneous in time for the level and partially homogeneous in the space. Local limit theorem for the first passage time follows from this theorem as a corollary.

Key words: Markov chain, limit theorem.

1. Introduction. Let $X = \{X_n = X(u,n),\ n \geqslant 0\}$ be a homogeneous in time Markov chain with values on the real line $R = (-\infty, \infty)$, with the initial value $u \equiv \equiv X(u,0) = X_0$ and transition probability

$$P(u,B) = (X_1 \in B),$$

where $X_1 = X(u,1)$ and let $B \in \beta(R)$ be a σ-algebra of Borel sets in R.

Recently there appeared many papers ([1],[2],[3],[5]), where in some boundary value problems for random walks described by Markov's chain, are studied. In these papers, a linear first passage time of the form

$$\tau_c = \inf\{n \geqslant 0 : \quad X_n \geqslant c\},$$

is considered, here $c \geqslant 0$ and we assume that $\inf\{\oslash\} = \infty$.

The role and importance of the first passage moment τ_c in the theory of boundary problems for random walks and also in applied problems of probability theory and mathematical statistics are explained in the papers [1, 2, 5, 9, 10, 11].

In the present paper we prove a theorem on asymptotic behavior of joint distribution of overshoot $R_c = X_{\tau_c} - c$ and the first passage moment τ_c of the form (1) of the Markov chain X. A local limit theorem follows from this theorem as a corollary under which we understand any statement on the fact that under some conditions there exists a function $P(n,c)$ such that $P(\tau_c = n) = P(n,c)(1 + o(1))$ as $c \to \infty$ $(n = n(c) \to \infty)$.

In particular, there is revealed the effect that when partial homogeneity property is satisfied in the Markov chain the asymptotics of probability $P(\tau_c = n)$ coincides with the asymptotics of the same probability for a case of ordinary process of summation of independent identically distributed random variables with positive mean value and finite variance. The property of asymptotic independence of the overshoot R_c and the first passage time τ_c is also revealed for a partially homogeneous Markov chain in the space, that for ordinary random walk is proved in the papers [9,10].

Notice that asymptotic behavior of probability $P(\tau_c = n)$ in the case when the Markov chain is described by the sums of independent identically distributed random values, is studied in the papers [6,7,8].

2. Conditions and denotation. By $\zeta(u) = X_1 - u$ we denote the jumps of the chain X from the state u for a step whose distribution is defined by the equality

$$P(u + \zeta(u) \in B) = P(u, B), \quad B \in \beta(R).$$

As in [2] for the chain X we assume that this is N-partially homogeneous (or simply partially homogeneous) in the space, i.e. for some $N \geqslant 0$ the transient probability $P(u, dv)$ for $u > N$ and $v > N$ depends only on the difference $v - u$. This means that the chain X behaves for $u > N$ as ordinary process of summation of independent random variables ζ_1, ζ_2 distributed as some random variable ζ, whose distribution on the set $(N - u, \infty)$ coincides with the projection of distribution of the jump $\zeta(u) = X(u, 1) - u$ of the Markov chain X.

Notice that the chain $Y(k)$ defined by the recurrent relations

$$Y(k+1) = \max(0, Y(k) + \zeta_{k+1}),$$

is 0-partially homogeneous. As is known ([1, 2, 11]) such chains describe the work of a number of queening systems.

In sequel, we denote $S_n = \sum_{k=j}^{n} \zeta_k$. We assume that the random variable ζ has the mean value $\mu = E\zeta > 0$ and variance $\sigma^2 = D\zeta < \infty$.

Notice that the paper is a continuation and complementation of the paper [3], where integral limit theorems are studied for the first passage time τ_c of a wider class of the so called asymptotically homogeneous (in time and in space) with drift of Markov's chains, when $E\zeta_n(u) \to \mu \in R$ as $u, n \to \infty$ (here $\zeta_n(x)$ is a jump from the state u at time n).

Let $B_0 \supseteq B_1 \supseteq \ldots$ be some non-increasing sequence of sets in R. As in [4] we say that the chain X_n is asymptotically homogeneous in time and space (in the direction of the set B_n), if the distribution of the jump $\zeta_n(u)$ weakly converges to the distribution of some random variable ζ as $u \to \infty$ uniformly with respect to $u \in B_n$.

Denote the characteristic functions of the random variables $\zeta_n(u) - \mu$ and $\zeta - \mu$ by

$$\varphi_n(\lambda, u) = Ee^{i\lambda(\zeta_n(u) - \mu)}$$

and

$$\varphi_n(\lambda) = Ee^{i\lambda(\zeta - \mu)}, \quad \lambda \in R.$$

3. Formulation and proof of the main result.

Theorem 1. *Let a homogeneous in time Markov chain X with the initial value $u = X_0$ be partially homogeneous in the space $(0 \leqslant N < u < \infty)$, and let a random variable ζ have a non-lattice distribution with $\mu = E\zeta > 0$ and $0 < \sigma^2 = D\zeta < \infty$.*

If $n = n(c) = \dfrac{c}{\mu} + \theta(c)\sqrt{c/\mu}$, $\theta(c) \to \theta \in R$ as $c \to \infty$ then

$$P(\tau_c = n,\ R_c \leqslant x) \sim \frac{\mu\varphi\left(\dfrac{\theta}{\sigma}\mu\right)}{\sigma\sqrt{n}} H(x), \quad c \to \infty$$

uniformly with respect to x, $0 < \delta \leqslant x < \infty$ and θ from a bounded set in R, where

$$H(x) = \frac{1}{ES_{\tau_+}} \int_0^x P\left(S_{\tau_+} > y\right) dy$$

and $\tau_+ = \inf\{n \geqslant 1 \cdot S_n > 0\}$.

Corollary 1. *(Local limit theorem). Under the conditions of the theorem, we have*

$$P(\tau_c = n) \sim \frac{\mu\varphi\left(\theta\dfrac{\mu}{\sigma}\right)}{\sigma\sqrt{n}}, \quad c \to \infty$$

uniformly with respect to θ from a bounded set R.

Corollary 2. *Under the conditions of the theorem as $c \to \infty$ we have*

$$P(R_c \leqslant x\,|\tau_c = n|) \to H(x), \quad x > 0.$$

Corollary 2 shows that for a Markov chain partially homogeneous in the space of the quantities of the overshoot R_c and the first passage time τ_c are asymptotically independent ([9,10]).

We need the following facts formulated in the form of lemmas.

Lemma 1. *Let a Markov's chain be homogeneous in time and in space, and "limit jump" ζ have finite mean value $\mu = E\zeta > 0$ and variance $\sigma^2 = D\zeta$.*

Assume that the distribution of random variable ζ is non-lattice and the following conditions are satisfied:

1) The central limit theorem, i.e. $\frac{X_n - n\mu}{\sigma\sqrt{n}} \Longrightarrow N(0,1)$ as $n \to \infty$ holds for the chain X.

2) For some non-increasing sequence $B_0 \supseteq B_1 \supseteq \ldots$ of sets in R it holds as $n \to \infty$

$$P\left(X_k \notin B_k \quad \text{for some} \quad k \geqslant n\right) = o\left(1/\sqrt{n}\right).$$

3) For any $a > 0$

$$\sup_{x \in B_n,\ \lambda \in [-a,a]} \left|\varphi_n(\lambda, x) - \varphi(\lambda)\right| o(1/n),\ n \to \infty.$$

Then for any $r > 0$

$$P(X_n \in (x, x+r)) = \frac{r}{\sigma\sqrt{2\pi n}} e^{-\dfrac{(x-n\mu)^2}{2\sigma^2 n}} + o\left(1/\sqrt{n}\right)$$

uniformly with respect to $x \in R$.

The statement of this lemma follows from the central local limit theorem for Markov's chain [4].

Lemma 2. *Let the conditions of Theorem 1 be satisfied. Then for any* $r > 0$

$$P\left(X_n \in (x, x+r)\right) = \frac{r}{\sigma\sqrt{2\pi n}} e^{-\dfrac{(x-n\mu)^2}{2\sigma^2 n}} + o\left(1/\sqrt{n}\right)$$

uniformly with respect to $x \in R$.

This lemma follows from Lemma 1, since in the conditions of Theorem 1 the conditions 1),2) and 3) of Lemma 1 (see also [4]) are satisfied.

By means of Lemma 2 we prove the following lemma that play an important role in the proof of the theorem.

Assume for $n \geqslant 1,\ y \in R$ and $k \geqslant 1$

$$Q_{n,k,h}\left(dx_{1,\ldots,}dx_k \mid \quad y = P\left(\zeta_1 \in dx_1, .., \zeta_k \in dx_k\right)\right| \quad X_n \in (y, y+h)) .$$

This is a conditional distribution of random variables $\zeta_1 ..., \zeta_k$ distributed as a random variable $\zeta\left(\zeta\left(u\right)\right)$ given that $X_n \in (y, y+h)$.

Lemma 3. *Let the conditions of Theorem 1 be satisfied. Then*

1) for each k *the conditional distribution* $Q_{n,k,h}\left(dx_{1,\ldots,}dx_k/y\right)$ *weakly converges as* $n \to \infty$ *to an unconditional distribution of random variables* $\zeta_1 ..., \zeta_k$ *and their convergence is fulfilled uniformly with respect to* $y,\ y - n\mu = O\left(\sqrt{n}\right)$ *and* h *from a bounded set* B *in* $(0, \infty)$, $0 < \inf B \leqslant \sup B < \infty$.

2) For any number $\delta \in (0,1)$ *there exists a constant in* $M = M(\delta)$ *such that for all* $y,\ y - n\mu = O\left(\sqrt{n}\right)$ *and* $h \in B$ *it holds*

$$Q_{n,k,h}\left(dx_{1,\ldots,}dx_k/y\right) \leqslant MP\left(\zeta \in dx_{1,\ldots,}\zeta_k \in dx_k\right).$$

Proof.

$$Q_{n,k,h}\left(dx_{1,\ldots,}dx_k/y\right) = \frac{P\left(\zeta \in dx_{1,\ldots,}\zeta_k \in dx_k,\ X_n \in (y, y+n)\right)}{P\left(X_n \in (y, y+n)\right)} =$$

$$= \frac{P\left(X_{n-k} \in \left(y - \sum\limits_{i=1}^{k} x_i, y - \sum\limits_{i=1}^{k} x_i + h\right) \mid \quad \zeta_1 \in dx_{1,\ldots,}\zeta_k \in dx_k\right)}{P\left(X_n \in (y, y+h)\right)} \times \qquad (1)$$

$$\times P\left(\zeta_1 \in dx_{1,\ldots,}\zeta_k \in dx_k\right) = \frac{P\left(X_{n-k} \in \left(y - \sum\limits_{i=1}^{k} x_i, y - \sum\limits_{i=1}^{k} x_i + h\right)\right)}{P\left(X_n \in (y, y+h)\right)} \times$$

$$\times P\left(\zeta_1 \in dx_1, \ldots, \zeta_k \in dx_k\right)$$

Lemma 2 yields

$$P\left(X_n \in (y, y+h)\right) \sim \frac{h}{\sigma\sqrt{n}} \varphi\varphi\left(\frac{y - n\mu}{\sigma\sqrt{n}}\right) \qquad (2)$$

uniformly with the respect to $y,\ y - n\mu = O\left(\sqrt{n}\right)$ where $\varphi(x)$ is the density of normal distribution with parameters (0,1).

By relation (2) it is easy to show that for the fixed k and $x \in R$

$$\lim_{n\to\infty} \frac{P\left(X_{n-k} \in (y-x, y-x+h)\right)}{P\left(X_n \in (y, y+h)\right)} = 1 \tag{3}$$

uniformly with respect to y, $y - n\mu = O\left(\sqrt{n}\right)$ and h from a bounded set in $(0, \infty)$.

Statement 1) of Lemma 3 follows from (1),(2) and (3).

In order to prove statement 2) of the lemma it suffices to show that as $n \to \infty$

$$\sup_{1\leqslant k\leqslant n(1-\delta)} \frac{P\left(X_{n-k} \in (y-x, y-x+h)\right)}{P\left(X_n \in (y, y+h)\right)} = O\left(1\right) \tag{4}$$

for the fixed x and y, $y - n\mu = O\left(\sqrt{n}\right)$ and h from a bounded set $B \subset (0, \infty)$.

Indeed, relation (4) follows from (2) and the following estimations

$$\sup_{h\in B,\ y:|y-n\mu|\leqslant c\sqrt{n}} \sqrt{n}P\left(X_n \in (y, y+h)\right) < \infty,$$

$$\inf_{h\in B,\ y:|y-n\mu|\leqslant c\sqrt{n}} \sqrt{n}P\left(X_n \in (y, y+h)\right) > 0$$

and

$$\sup_{1\leqslant k\leqslant n(1-\delta)} \sqrt{\frac{n}{n-k}} \leqslant \sqrt{1/\delta}.$$

We need the following lemma.

Lemma 4. *Let all the conditions of the theorem be satisfied. Then for any $\varepsilon > 0$ there exists an integer q_1 such that for sufficiently large c and for all r, h and x from a bounded set in R it holds*

$$I\left(c\right) = P\left(X_n - X_{n-i} \leqslant r, \quad \exists i \in (q_1, n] \quad | \quad X_n \in c + \Delta\,\right) < \varepsilon,$$

where $\Delta = (x, x+h)$.

Proof. We have $I\left(c\right) = I_1\left(c\right) + I_2\left(c\right)$ where

$$I_1\left(c\right) = P\left(X_n - X_{n-i} \leqslant r, \quad \exists i \in (q_1, n] \quad | \quad X_n \in c + \Delta\,\right) < \varepsilon$$

and

$$I_2\left(c\right) = P\left(X_n - X_{n-i} \leqslant r, \quad \exists i \in (n\delta, n) \quad | \quad X_n \in c + \Delta\,\right)$$

At first we estimate $I_2\left(c\right)$. Assuming $n - i = j$ we have

$$I_2\left(c\right) = P\left(X_j - X_n - r, \quad \exists i \in \ \left[1\left(1-\delta\right)n\right)/X_n \in c + \Delta\right) \leqslant$$

$$\leqslant \left(X_j \geqslant c - x - r, \quad \exists j \in \ \left[1\left(1-\delta\right)n\right)/X_n \in c + \Delta\right).$$

Hence for $0 < \delta < 1/2$ and $a = c - x - r$ from statement 2 of Lemma 1 we get

$$I_2\left(c\right) \leqslant MP\left(\tau_a \leqslant \left(1-\delta\right)n\right). \tag{5}$$

Further by the strong law of large numbers, for the process τ_c [3] we have

$$\frac{\tau_c}{c} \overset{n-1}{\to} \frac{1}{\mu}.$$

Therefore

$$P\left(\tau_a \leqslant \left(1-\delta\right)n\right) \to 0 \quad \text{as} \quad c \to \infty,$$

since

$$n = n(c) \sim \frac{c}{\mu}.$$

Consequently, it follows from (5) that for sufficiently large c

$$I_2(c) < \frac{c}{2}.$$

Now, let us estimate $I_1(c)$. Notice that it follows from the partial homogeneity property of the chain X_n that in the domain $u > N$ the difference $X_n - X_{n-i}$ is distributed as the sum $S_i = \sum_{i=1}^{i} \zeta_i$ of the steps of the Morkov chain X for i steps. Therefore, from statement 2 of Lemma 3 we have

$$I_1(c) = P(S_i \leqslant r, \quad \exists i \in [q_1, n\delta) / X_n \in c + \Delta) \leqslant MP(S_i \leqslant r, \ \exists i \geqslant q_1). \qquad (6)$$

The strong law of large numbers for the Markov chain [4] holds in the conditions of theorem 1. Therefore, it follows from (6) that $I_1(c) < \varepsilon/2$ is fulfilled for sufficiently large q_1 and c.

Thus, the statement of Lemma 3 is proved.

Proof of the theorem 1. Divide the interval $(0, r]$ into m equal parts and let

$$\Delta_k = \left(\frac{k-1}{m}r, \ \frac{k}{m}\right], \quad k = \overline{1, m}.$$

By the total probability formula we have

$$P(\tau_c = n, \quad R_c \leqslant r) = \sum_{k=1}^{m} P(\tau_c = n / X_n \in c + \Delta_k) P(X_n \in c + \Delta_k). \qquad (7)$$

Considering $\{\tau_c > n\} \subset \{X_n \leqslant c\}$ for sufficiently large c we have

$$P(\tau_c = n \mid \quad X_n \in c + \Delta_k) = P(\tau_c \geqslant n \mid \quad X_n \in c + \Delta_k) =$$
$$= P(X_i \leqslant c, \quad 1 \leqslant i < n \mid \quad X_n \in c + \Delta_k) =$$
$$= P(X_n - X_i \geqslant X_n - c, \quad 1 \leqslant i < n \mid \quad X_n \in c + \Delta_k) =$$
$$= P(X_n - X_i \geqslant X_n - c, \quad 1 \leqslant i < n \mid \quad X_n \in c + \Delta_k) =$$
$$= P(S_i \geqslant X_n - c, \quad 1 \leqslant i < n \mid \quad X_n \in c + \Delta_k).$$

Hence it is easy to see that

$$P\left(S_i \geqslant \frac{k}{m}r, \quad 1 \leqslant i < n \,| X_n \in c + \Delta_k\right) \leqslant$$
$$\leqslant P(\tau_c = n \mid \quad X_n \in c + \Delta_k) \leqslant P\left(S_i \geqslant \frac{k-1}{m}r, \quad 1 \leqslant i < n \mid \quad X_n \in c + \Delta_k\right). \qquad (8)$$

From (8) and equality (7) we get

$$\sum_{k=1}^{m} P\left(S_i \geqslant \frac{k}{m}r, \quad 1 \leqslant i < n \mid \quad X_n \in c + \Delta_k\right) P(X_n \in c + \Delta_k) \leqslant$$
$$\leqslant P(\tau_c = n, \quad R_c \leqslant r) \leqslant \sum_{k=1}^{m} P(X_n \in c + \Delta_k) \times \qquad (9)$$

$$\times P\left(S_i \geqslant \frac{k-1}{m}r, \quad 1 \leqslant i < n \,|\quad X_n \in c+\Delta_k\right).$$

Further, it follows from statement 1 of Lemma 2 that for k and fixed $p \geqslant 1$

$$\lim_{n\to\infty} P\left(S_i \geqslant x, \quad 1 \leqslant i \leqslant p \,|\quad X_n \in c+\Delta_k\right) = P\left(S_i \geqslant x, \quad 1 \leqslant i \leqslant p\right). \tag{10}$$

Since $E\zeta_1 = \mu > 0$, it follows from the strong law of large numbers that for any $\varepsilon > 0$ there exists a sufficiently large integer q_2

$$P\left(S_i \leqslant x, \quad \exists i > q_2\right) < \varepsilon \tag{11}$$

The following bilateral inequalities follow from Lemma 4 and estimation (11) for $q = \max(q_1, q_2)$:

$$P\left(S_i \geqslant x, 1 \leqslant i \leqslant q \,|X_n \in c+\Delta_k\right) - \varepsilon \leqslant P\left(S_i \geqslant x, 1 \leqslant i \leqslant n \,|X_n \in c+\Delta_k\right) \leqslant \tag{12}$$

$$\leqslant P\left(S_i \geqslant x, 1 \leqslant i \leqslant q \,|X_n \in c+\Delta_k\right)$$

and

$$P\left(S_i \geqslant x, 1 \leqslant i \leqslant q\right) - \varepsilon \leqslant P\left(S_i \geqslant x,\ i \geqslant 1\right) \leqslant P\left(S_i \geqslant x,\ 1 \leqslant i \leqslant q\right) \tag{13}$$

It follows from (10),(12) and (13) for $q = p$ that

$$P\left(S_i \geqslant x,\ i \geqslant 1\right) - 2\varepsilon \leqslant P\left(S_i \geqslant x, 1 \leqslant i < n \,|X_n \in c+\Delta_k\right) \leqslant$$

$$\leqslant P\left(S_i \geqslant x,\ i \geqslant 1\right) + 2\varepsilon.$$

From (9) and the last inequality we have

$$\sum_{k=1}^{m}\left(P\left(S_i \geqslant \frac{k}{m}x,\ i \geqslant 1\right) - 2\varepsilon\right) P\left(X_n \in c+\Delta_k\right) \leqslant$$

$$\leqslant P\left(\tau_c = n, R_c \leqslant x\right) \sum_{k=1}^{m}\left(P\left(S_i \geqslant \frac{k-1}{m}x,\ i \geqslant 1\right) + 2\varepsilon\right) P\left(X_n \in c+\Delta_k\right). \tag{14}$$

By lemma 1,

$$P\left(X_n \in c+\Delta_k\right) \sim \frac{\frac{x}{m}}{\sigma\sqrt{n}}\varphi\left(\frac{c-n\mu}{\sigma\sqrt{u}}\right) \tag{15}$$

as $c \to \infty$.

In view of $n = \dfrac{c}{\mu} + \theta(c)\sqrt{c/\mu}$, $\theta(c) \to \theta \in R$ as $c \to \infty$, from (15) we find

$$P\left(X_n \in c+\Delta_k\right) \sim \frac{x/m}{\sigma\sqrt{n}}\varphi\left(\frac{\theta\mu}{\sigma}\right) \text{ as } c \to \infty. \tag{16}$$

Substituting (16) into (14) for sufficiently large c we have

$$\varphi\left(\frac{\mu}{\sigma}\theta\right)(1-\varepsilon)\sum_{k=1}^{m}\frac{x}{m}P\left(T \geqslant \frac{k}{m}x\right) - 2\varepsilon,$$

$$\sigma\sqrt{n}P\left(\tau_c = n, R_c \leqslant x\right) \leqslant \varphi\left(\frac{\mu}{\sigma}\theta\right)(1+\varepsilon)\sum_{k=1}^{m}P\left(T \geqslant \frac{k-1}{m}x\right) + 2\varepsilon,$$

where $T = \inf\limits_{i>1} S_i$.

As $m \to \infty$ and $\varepsilon \to \infty$ the left and right hand sides of the last inequality tend to the limit $\varphi\left(\frac{\theta\mu}{\sigma}\right)\int\limits_0^x P(T \geqslant y)\,dy$.

Thus

$$P(\tau_c = n, \quad R_c \leqslant x) \sim \frac{\varphi\left(\frac{\theta\mu}{\sigma}\right)}{\sigma\sqrt{n}} \int\limits_0^x P(T \geqslant y)\,dy.$$

In order to get the affirmation of the proved theorem from the last relation it sufficies to note the following equality whose proof is in the paper [9]:

$$P(T \geqslant y) = \frac{\mu}{E\left(S_{\tau_+}\right)} P\left(S_{\tau+} \geqslant y\right),$$

where $\tau_+ = \inf\{i \geqslant 1; S_i > 0\}$ is the first stair moment for the sum $S_i = \sum\limits_{j=1}^{i} \zeta_j$.

Notice that Corollary 1 directly follows from the statement of the theorem as $x \to \infty$ and Corollary 2 follows from the theorem and Corollary 1.

Remark. Statement of the theorem and its corollaries for an ordinary random walk are contained in the papers [9,10]. Notice that the studying of the boundary value problems for Markov's partially homogeneous chain in many respects can be realized by means of the results of the theory of summation of independent random variables ([2,9,10,12]).

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АСИМПТОТИЧНА ПОВЕДІНКА СУМІСНОГО РОЗПОДІЛУ ПЕРЕСКОКУ І МОМЕНТУ ПЕРШОГО ВИХОДУ ЗА РІВЕНЬ ЛАНЦЮГА МАРКОВА

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Доведено теорему про асимптотичну поведінку сумісного розподілу перескоку і моменту першого виходу за рівень ланцюга Маркова. Як наслідок отримується локальна гранична теорема для моменту першого виходу за рівень ланцюга Маркова.

Ключові слова: ланцюг Маркова, гранична теорема.

АСИМПТОТИЧЕСКОЕ ПОВЕДЕНИЕ СОВМЕСТНОГО РАСПРЕДЕЛЕНИЯ ПЕРЕСКОКА И МОМЕНТА ПЕРВОГО ВЫХОДА ЗА УРОВЕНЬ ЦЕПИ МАРКОВА

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Доказана теорема о асимптотическом поведении совместного распределения перескока и момента первого выхода за уровень цепи Маркова. Как сладствие получена локальная предельная теорема для момента первого выхода за уровень цепи Маркова.

Ключевые слова: цепь Маркова, предельная теорема.

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