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PROCEDURE OF TRANSFORMING THE MINIMAXED PROBABILITY DISTRIBUTION DUE TO TRACING AN OBJECT FOR EVALUATING THE OBJECT FACTOR WITH A FINITE SET OF ITS THEORETICAL VALUES

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Estimation of an object factor, assigned to a finite set of theoretical values, is stated. The basic object factor evaluation is made via the probability distribution per the minimax risk. There is advanced the procedure of transformation of the minimaxed probability distribution due to tracing the object. The finally transformed probability distribution is accepted only if it is sufficiently close to the accumulated statistical distribution.

Key words: an object factor, theoretical values, probability distribution, minimax risk, a second player optimal strategy, expectation, classification rule, statistical distribution, sufficiently close distributions.

1. POSING A QUESTION

When exploring the object, there springs a lot of its factors to be observed and described mathematically. An object factor may be assigned to several values simultaneously, being at that also depending upon a series of inner variables, such as time, location, energy consumption and so forth. One of the questions for identifying and tracing the object properly is in evaluating its factors, and this needs an adequate probability distribution (PD) over the aggregate of the factor values.

2. ANALYSIS OF MEANS

Every factor of the object should be estimated, wherever the object exploration process is. In the very beginning of that process there is poor information about PD over the aggregate of the factor values [1, 2], although at least a few ways of estimating the factor always exist. Within those ways a primitive PD may be determined via minimizing assuredly the absolute deviations among the theoretically fixed factor values [3, 4]. Any other PD is hard to substantiate without watching the object for a short or even long while. And if to accept the equiprobable distribution (EPD) for a basis, then it will yield to the primitive minimaxed PD [5, 6]. But even with the basic PD there ought to be advanced a procedure of evolving this PD into such PD over the theoretically fixed factor values, that the evolved PD would be as sufficiently close as practically required to the statistical PD, obtained through the procedure of tracing the object.

3. TARGET

For the start of the procedure of tracing the object there is going to be considered a finite aggregate of the factor values, over which no PD is defined due to that any PD is equipossible. Each value depends upon a series of its inner variables, being permanent for all the theoretically fixed factor values. However, as the tracing procedure starts with its observations and measurements over the object, the statistical PD is being progressively obtained. The target is to evolve PD from the basic one through a given number of steps towards the finally transformed PD, which can be used for evaluating the object factor under its inner variables.

4. ADVANCING THE PROCEDURE OF THE MINIMAXED PD TRANSFORMATION

Let

$$\{w_i(\mathbf{X})\}_{i=1}^{N} \text{ at } N \in \mathbb{N}\setminus\{1\} \tag{1}$$

be the set of the object factor w values under its inner variables, grouped into the point

$$\mathbf{X} = [x_1 \quad x_2 \quad \cdots \quad x_{M-1} \quad x_M] \in \mathbb{R}^M \tag{2}$$

with

$$x_m \in X_m \subset \mathbb{R} \text{ by } m = \overline{1, M} \text{ at } M \in \mathbb{N}. \tag{3}$$

If to have hit the spoken above target, the object factor w uncertainty, generated by the set (1), can be reduced into the evaluation

$$w(\mathbf{X}) \ \forall\, x_m \in X_m \subset \mathbb{R} \text{ for } m = \overline{1, M} \tag{4}$$

by the finally transformed PD.

The basic PD for the set (1) for (2) with (3) can be estimated per the minimax risk [3] only. Particularly, minimizing Euclidean distances

$$k_{ij}(\mathbf{X}) = |w_i(\mathbf{X}) - w_j(\mathbf{X})| \text{ by } i = \overline{1, N} \text{ and } j = \overline{1, N} \tag{5}$$

as elements of the matrix

$$\mathbf{K}(\mathbf{X}) = [k_{ij}(\mathbf{X})]_{N \times N} \tag{6}$$

in the game

$$\left\langle \{c_i\}_{i=1}^{N}, \{s_j\}_{j=1}^{N}, \mathbf{K}(\mathbf{X}) \right\rangle \tag{7}$$

for every point (2) with (3), where the first player pure strategy c_i means that the i-th object factor value $w_i(\mathbf{X})$ from the set (1) actually prevails under its inner variables (2), whereas the second player as investigator practically selects the j-th object factor value $w_j(\mathbf{X})$ via its pure strategy s_j. In the game (7) with the matrix (6) of elements (5) the investigator has its optimal strategy

$$\breve{\mathbf{Q}}(\mathbf{X}) = [\breve{q}_1(\mathbf{X}) \quad \breve{q}_2(\mathbf{X}) \quad \cdots \quad \breve{q}_{N-1}(\mathbf{X}) \quad \breve{q}_N(\mathbf{X})] \in \breve{\mathcal{Q}}(\mathbf{X}) \subset$$

$$\subset \mathcal{Q} = \left\{ \mathbf{Q} = [q_1 \quad q_2 \quad \cdots \quad q_{N-1} \quad q_N] \in \mathbb{R}^N : q_j \in [0; 1] \ \forall\, j = \overline{1, N}, \sum_{j=1}^{N} q_j = 1 \right\} \tag{8}$$

from the set of all the second player optimal strategies

$$\breve{\mathcal{Q}}(\mathbf{X}) = \arg \min_{\mathbf{Q} \in \mathcal{Q}} \max_{\mathbf{P} \in \mathcal{P}} \left(\mathbf{P} \cdot \mathbf{K}(\mathbf{X}) \cdot \mathbf{Q}^{\mathrm{T}} \right) \tag{9}$$

under inner variables (2) for the $(N-1)$-dimensional fundamental simplex

$$\mathcal{P} = \left\{ \mathbf{P} = [p_1 \quad p_2 \quad \cdots \quad p_{N-1} \quad p_N] \in \mathbb{R}^N : p_i \in [0; 1] \ \forall\, i = \overline{1, N}, \sum_{i=1}^{N} p_i = 1 \right\} \tag{10}$$

of all possible PD over N different values, though conditions of generating those PD cannot be structured or forecasted [3, 7], what is personified by the first player.

Having minimized the distances (5) over $(N-1)$-dimensional fundamental simplex $\mathcal{Q}$ in (8) under non-forecasted conditions as the maximized distances (5) over $(N-1)$-

dimensional fundamental simplex (10), the components of the optimal strategy (8) may be used to determine the minimaxed expectation

$$\tilde{w}^{\langle 0\rangle}(\mathbf{X}) = \sum_{j=1}^{N} w_j(\mathbf{X})\breve{q}_j(\mathbf{X}) \tag{11}$$

of the object factor w. Nevertheless, through the procedure of tracing the object in the given number of steps $U \in \mathbb{N}$ the minimaxed PD in (8) should be evolved into some PD, which would be sufficiently close to the statistical PD.

May on the u-th step of tracing the object under inner variables (2)

$$\mathbf{X}_A = \begin{bmatrix} x_1^{\langle t_1\rangle} & x_2^{\langle t_2\rangle} & \cdots & x_{M-1}^{\langle t_{M-1}\rangle} & x_M^{\langle t_M\rangle} \end{bmatrix} \in \mathbb{R}^M \text{ at } A = \{t_m\}_{m=1}^{M} \text{ by } \left\{x_m^{\langle t_m\rangle}\right\}_{t_m \in T_m} \subset X_m \tag{12}$$

of its factor w there be measured the value

$$w^{\langle u\rangle}(\mathbf{X}_A) = w^{\langle u\rangle}\left(\left\{x_m^{\langle t_m\rangle}\right\}_{m=1}^{M}\right) \text{ by } u = \overline{1, U} \text{ at } U \in \mathbb{N} \tag{13}$$

of this factor. Every set of indexes in $\{T_m\}_{m=1}^{M}$ is finite, as tracing the object generally cannot be continual, and so the m-th inner variable of the factor w varies within the finite set $\left\{x_m^{\langle t_m\rangle}\right\}_{t_m \in T_m}$. The procedure of evolving PD in (8), giving the minimaxed expectation (11), into PD

$$\tilde{\mathbf{Q}}(\mathbf{X}_A) = \begin{bmatrix} \tilde{q}_1(\mathbf{X}_A) & \tilde{q}_2(\mathbf{X}_A) & \cdots & \tilde{q}_{N-1}(\mathbf{X}_A) & \tilde{q}_N(\mathbf{X}_A) \end{bmatrix} \tag{14}$$

with

$$\tilde{q}_j(\mathbf{X}_A) \in [0;\, 1] \text{ by } \sum_{j=1}^{N} \tilde{q}_j(\mathbf{X}_A) = 1 \tag{15}$$

for every $t_m \in T_m$ of the set $A = \{t_m\}_{m=1}^{M}$ in (12), is starting as the following. Initially

$$\tilde{q}_j^{\langle 0\rangle}(\mathbf{X}_A) = \breve{q}_j(\mathbf{X}_A) \quad \forall\ j = \overline{1, N} \tag{16}$$

and then the j_*-th probability is corrected as

$$\tilde{q}_{j_*}^{\langle u\rangle}(\mathbf{X}_A) = \alpha \tilde{q}_{j_*}^{\langle u-1\rangle}(\mathbf{X}_A) \text{ at } j_* \in \left\{\overline{1, N}\right\} \tag{17}$$

with some coefficient

$$\alpha \in \left(1;\, \frac{1}{\tilde{q}_{j_*}^{\langle u-1\rangle}(\mathbf{X}_A)}\right], \tag{18}$$

whereupon other probabilities

$$\tilde{q}_j^{\langle u\rangle}(\mathbf{X}_A) = \beta \tilde{q}_j^{\langle u-1\rangle}(\mathbf{X}_A) \text{ for } j \in \left\{\overline{1, N}\right\} \setminus \{j_*\} \tag{19}$$

are corrected with the coefficient

$$\beta = \frac{1 - \alpha \tilde{q}_{j_*}^{\langle u-1\rangle}(\mathbf{X}_A)}{\sum\limits_{j \in \left\{\overline{1, N}\right\} \setminus \{j_*\}} \tilde{q}_j^{\langle u-1\rangle}(\mathbf{X}_A)}, \tag{20}$$

using a classification rule [8]

$$j_* \in \arg\min_{j=\overline{1, N}} \left| w^{\langle u\rangle}(\mathbf{X}_A) - w_j(\mathbf{X}_A) \right| \tag{21}$$

with the factor w expectation

$$\tilde{w}^{\langle u-1\rangle}\left(\mathbf{X}_A\right)=\sum_{j=1}^{N} w_j\left(\mathbf{X}_A\right)\tilde{q}_j^{\langle u-1\rangle}\left(\mathbf{X}_A\right) \tag{22}$$

after the $(u-1)$-th correction for every $t_m \in T_m$ of the set $A=\{t_m\}_{m=1}^{M}$ in (12).

It is clear the correction coefficient (18) should be chosen due to the distance between $w^{\langle u\rangle}\left(\mathbf{X}_A\right)$ and (22), being a nondecreasing function

$$\alpha=\alpha\left(\left|w^{\langle u\rangle}\left(\mathbf{X}_A\right)-\tilde{w}^{\langle u-1\rangle}\left(\mathbf{X}_A\right)\right|\right), \tag{23}$$

that is

$$\alpha\left(z_1\right)\leqslant\alpha\left(z_2\right) \text{ for } z_1<z_2 \tag{24}$$

by any value (13) and any (22) for all possible points (12). The nondecreasing function (23) as (24) should be defined carefully or else the final probabilities

$$\tilde{q}_j\left(\mathbf{X}_A\right)=\tilde{q}_j^{\langle U\rangle}\left(\mathbf{X}_A\right) \quad \forall\ j=\overline{1, N} \tag{25}$$

for every $t_m \in T_m$ of the set $A=\{t_m\}_{m=1}^{M}$ in (12) are running to fail conditions

$$\left|\tilde{q}_j\left(\mathbf{X}_A\right)-\eta_j\left(\mathbf{X}_A\right)\right|\leqslant\delta \quad \forall\ j=\overline{1, N} \tag{26}$$

and

$$\sum_{j=1}^{N}\left(\tilde{q}_j\left(\mathbf{X}_A\right)-\eta_j\left(\mathbf{X}_A\right)\right)^2\leqslant\rho \tag{27}$$

for sufficient closeness PD (14) to the statistical PD from probabilities $\left\{\eta_j\left(\mathbf{X}_A\right)\right\}_{j=1}^{N}$. These statistical probabilities are accumulated due to the classification rule (21), where

$$\eta_j\left(\mathbf{X}_A\right)=\eta_j^{\langle U\rangle}\left(\mathbf{X}_A\right) \quad \forall\ j=\overline{1, N} \tag{28}$$

after

$$\eta_{j_*}^{\langle u\rangle}\left(\mathbf{X}_A\right)=\frac{(u-1)\eta_{j_*}^{\langle u-1\rangle}\left(\mathbf{X}_A\right)+1}{u} \text{ at } j_*\in\left\{\overline{1, N}\right\} \tag{29}$$

and

$$\eta_j^{\langle u\rangle}\left(\mathbf{X}_A\right)=\frac{(u-1)\eta_j^{\langle u-1\rangle}\left(\mathbf{X}_A\right)}{u} \text{ for } j\in\left\{\overline{1, N}\right\}\backslash\left\{j_*\right\} \tag{30}$$

by $u=\overline{1, U}$. Hence, every evaluation

$$\tilde{w}\left(\mathbf{X}_A\right)=\sum_{j=1}^{N} w_j\left(\mathbf{X}_A\right)\tilde{q}_j\left(\mathbf{X}_A\right) \tag{31}$$

of the object factor w under its inner variables (12) by PD (14) is valid only if conditions (26) and (27) are true for some $\delta>0$ and $\rho>0$. Consequently, the procedure of the minimaxed PD (8) transformation into PD (14) satisfies the requirement of sufficient closeness PD (14) to the statistical PD (28) in the sense of making true (26) and (27). And if (26) or (27) is false then there should be continued the object tracing, where the nondecreasing function (23) will probably be refined, until for the increased U requirements (26) and (27) are true.

5. ILLATIVE SPOTS

When there is poor information about PD over the aggregate of the theoretically fixed factor values (1) of the object, being theoretically traced in the location (2) of the inner variables space $\prod_{m=1}^{M} X_m = \boldsymbol{\mathcal{X}}$, then the minimaxed PD (8) is based for PD transformation start-off. For start-off EPD is unacceptable, yielded [5, 6] to any PD from the minimaxed set (9). Note, that if the set (9) is non-singleton, then before starting to evolve the minimaxed PD the problem of selection the single element (8) must be solved [6]. Having defined some $\delta > 0$ and $\rho > 0$ for requirements (26) and (27) in the fixed location (12), measurements (tracing the object practically) in (13) with subsequent (16) – (24) for evolving the minimaxed PD (8) are carried out until the steps total is U or requirements (26) and (27) are true. Probabilities (25) with (15) of the evolved PD (14) are used for the object factor w evaluation (31) in the location (12). This lets reduce the object factor w uncertainty, generated by the set (1), into the evaluation in the form (4) as

$$w(\mathbf{X}_A) = \tilde{w}(\mathbf{X}_A) \;\; \forall\; \mathbf{X}_A \in \boldsymbol{\mathcal{X}} \tag{32}$$

by the finally transformed PD (14). Furthermore, the accumulated statistical probabilities (28) after (29) and (30) by $u = \overline{1, U}$ may be alternatively used for the object factor w evaluation

$$\tilde{w}_{\eta}(\mathbf{X}_A) = \sum_{j=1}^{N} w_j(\mathbf{X}_A)\eta_j(\mathbf{X}_A) \;\; \forall\; \mathbf{X}_A \in \boldsymbol{\mathcal{X}} \tag{33}$$

by the traced statistics. Both evaluations (32) as (31) and (33) are acceptable, being non-identical, though. And there remains a final spot to hit that the represented paper actually states acquiring not a single PD from N elements in (14), but a finite number of PD $\left\{ \left\{ \tilde{q}_j(\mathbf{X}_A) \right\}_{j=1}^{N} \right\}_{\mathbf{X}_A \in \boldsymbol{\mathcal{X}}}$ for the corresponding evaluations (32). A further-work problem is tied with fitting the nondecreasing function (23), whose efficient fit will decrease the steps total U (what accelerates the procedure of the minimaxed PD transformation) by the unalterable requirements (26) and (27).

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ПРОЦЕДУРА ПЕРЕТВОРЕННЯ МІНІМАКСНОГО РОЗПОДІЛУ ЙМОВІРНОСТЕЙ ЗГІДНО З ВІДСТЕЖУВАННЯМ ОБ'ЄКТА ДЛЯ ОЦІНЮВАННЯ ОБ'ЄКТОВОГО ФАКТОРА ЗІ СКІНЧЕННОЮ МНОЖИНОЮ ЙОГО ТЕОРЕТИЧНИХ ЗНАЧЕНЬ

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Викладено визначення об'єктового фактора, який приписаний до скінченної множини теоретичних значень. Базова оцінка об'єктового фактора виконується за допомогою розподілу ймовірностей згідно з мінімаксним ризиком. Подано процедуру перетворення розподілу ймовірностей, отриманого за правилом мінімаксу, відповідно до відстежування об'єкта. Остаточно перетворений розподіл імовірностей приймається, тільки якщо він достатньо близький до накопиченого статистичного розподілу.

Ключові слова: об'єктовий фактор, теоретичні значення, розподіл імовірностей, мінімаксний ризик, оптимальна стратегія другого гравця, очікуване значення, правило класифікації, статистичний розподіл, досить близькі розподіли.

ПРОЦЕДУРА ПРЕОБРАЗОВАНИЯ МИНИМАКСНОГО РАСПРЕДЕЛЕНИЯ ВЕРОЯТНОСТЕЙ В СООТВЕТСТВИИ С ОТСЛЕЖИВАНИЕМ ОБЪЕКТА ДЛЯ ОЦЕНИВАНИЯ ОБЪЕКТНОГО ФАКТОРА С КОНЕЧНЫМ МНОЖЕСТВОМ ЕГО ТЕОРЕТИЧЕСКИХ ЗНАЧЕНИЙ

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Излагается определение объектного фактора, приписываемого к конечному множеству теоретических значений. Базовая оценка объектного фактора осуществляется с помощью распределения вероятностей согласно минимаксному риску. Представляется процедура преобразования распределения вероятностей, полученного по правилу минимакса, в соответствии с отслеживанием объекта. Окончательно преобразованное распределение вероятностей принимается, только если оно достаточно близко к накопленному статистическому распределению.

Ключевые слова: объектный фактор, теоретические значения, распределение вероятностей, минимаксный риск, оптимальная стратегия второго игрока, ожидаемое значение, правило классификации, статистическое распределение, достаточно близкие распределения.