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MATHEMATICAL MODELS OF PLANE WAVE SCATTERING ON MULTILAYER IMPEDANCE STRUCTURES

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The method of constructing of electromagnetic waves diffraction on multilayer not perfectly conducting reflective structures had been built. The initial boundary-value problems for the Helmholtz equation with boundary conditions of the third kind had been reduced to a system of boundary integral equations of the first and second kind. The method of integral operators' parametric representations has been used in the derivation of integral equations. The method of discrete singularities or the method of mechanical quadrature can be used for numerical solution of obtained integral equations systems.

Key words: diffraction problems, multi-reflecting structure, boundary value problems, method of integral operators' parametric representations.

1. INTRODUCTION

Multilayer gratings, due to a special selection of the relations between the sizes of structure elements, produce a field with the necessary physical characteristics $[1,2]$. Modeling the excitation resonance structures the researches must take into account the absorption of the energy by structure. This leads to the creation of mathematical models that take into account the finite conductivity of materials $[3,4]$. The existence of the energy loss is often described by using the Shchukin-Leontovich boundary-value condition on the surface of not perfectly conducting screens:

$$[n\,E] = -Z_s \cdot [n[n\,H]]$$

where (E,H) – total electromagnetic field, Z_s -the surface impedance of the structure, n – the normal to the surface. Therefore, the construction of mathematical models of electromagnetic waves scattering on multilayer gratings with finite conductivity is an important problem for investigation.

One of the most effective ways to build mathematical models of electrodynamic systems is to use the method of parametric representations of integral transformations $[5,6]$. This method reduces the initial boundary value problem to an equivalent system of integral equations. The obtained systems of integral equations are solved numerically by using the method of discrete singularities $[7-9]$. This approach has proved its efficiency in solving of various electrodynamic problems $[9]$.

2. THE PROBLEM FORMULATION

Using the method of parametric representations of singular integral transformations will be shown on the example of this problem. Consider the following diffraction structure (see Fig. 1). There is an infinite screen in the plane $z = d_3$. Two layers of dielectrics lie on this screen. First dielectric layer with permittivity ε_1, fills the domain $d_2 < z < d_1$, the second dielectric layer with permittivity ε_2, fills the area $d_3 < z < d_2$. Between upper and lower dielectric layers and on the upper layer of dielectric are endless screens with apertures. Screens are made of material with finite conductivity. The half-plane z>d_1 with a dielectric permittivity ε_0=1 is above the described structure.

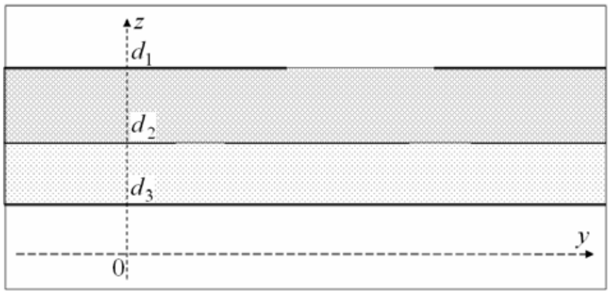

Fig. 1. A cross-section of the diffraction structure by the plane YOZ

Let us define the points set

$$L_i = \bigcup_{q=1}^{M_i} (\alpha_{i,q}, \beta_{i,q}), \quad -\infty < \alpha_{i,1} < \beta_{i,1} < \ldots < \alpha_{i,q} < \beta_{i,q} \ldots < \alpha_{i,M_i} < \beta_{i,M_i} < +\infty, \quad i = 1,2;$$

The sets L_i are the projection on the axis OY apertures in the plane $z = d_i$.

We introduce the definition of domains:

$$\Omega_0 = \{(y,z) \in R^2 \ | z > d_1 , y \in R\},$$

$$\Omega_i = \{(y,z) \in R^2 \ | d_{i+1} < z < d_i , y \in R\}, \quad i = 1,2.$$

The time dependence of the fields is given by the factor $e^{-i\omega t}$. A plane monochromatic H-polarized electromagnetic wave of unit amplitude is falling from the infinity on the top of the diffraction structure:

$$U_0(y,z) = H_{0,x}(y,z) = \exp(ik(y \cdot \sin\varphi - (z - d_1) \cdot \cos\varphi)), \quad k = \frac{\omega}{c}$$

where c – the speed of light in vacuum. It is necessary to find the total field $u(y,z) = H_x(y,z)$, appeared as a result of waves scattering on the considered structure. It satisfies the Helmholtz equation:

$$\Delta u + k^2 \varepsilon_i \cdot u = 0, \quad (y,z) \in \Omega_i, \quad (i = 0,1,2)$$

in the domains Ω_i, $(i = 0,\ldots,2)$, Meixner condition on edges, the scattered field - the difference between total and incident field satisfies the Sommerfeld radiation conditions.

In the case of H-polarization, the Shchukin-Leontovich boundary-value condition reduced to relations:

$$\frac{\partial u(y, d_3 + 0)}{\partial z} - h_2 \cdot u(y, d_3 + 0) = 0, \quad y \in R; \tag{1}$$

$$\frac{\partial u(y, d_i + 0)}{\partial z} - h_{i-1} \cdot u(y, d_i + 0) = 0, \quad y \in CL_i = R \setminus L_i, \ i = 1,2; \tag{2}$$

$$\frac{\partial u(y, d_i - 0)}{\partial z} + h_i \cdot u(y, d_i - 0) = 0, \quad y \in CL_i, \quad i = 1,2; \tag{3}$$

$$h_i = ik\varepsilon_i \, Z_s Z_0^{-1}, \ i = 0,1,2.$$

Objective: To construct a mathematical model of the H-polarized wave scattering problem on a multilayer impedance reflecting structure on the base of the boundary integral equations system.

3. DERIVATION OF BOUNDARY INTEGRAL EQUATIONS

Lets define function $U(y,z)$ - a solution of the auxiliary boundary-value problem of diffraction of plane monochromatic wave, defined by (1), on an infinite solid screen, which has not the ideal conductivity and which fills the plane $z = d_1$.

Field $U(y,z)$ has the form:

$$U(y,z) = \exp\{ik(y\cdot\sin\varphi - (z-d_1)\cdot\cos\varphi)\} + \\ + \frac{i\kappa\cos\varphi + h}{i\kappa\cos\varphi - h}\cdot\exp\{ik(y\cdot\sin\varphi + (z-d_1)\cdot\cos\varphi)\}$$

and the properties:

$$\frac{\partial U(y,d_1)}{\partial z} - h_0\cdot U(y,d_1) = 0, \quad y\in R;$$

$$U(y,d_1) = \frac{2i\kappa\cos\varphi}{i\kappa\cos\varphi - h}\cdot\exp(ik(y\cdot\sin\varphi)).$$

We are looking for the total field $u(y,z)$, appeared as a result of wave diffraction on the gratings, in the form: $u(y,z) = \delta_{0,i}\cdot U(y,z) + u_i(y,z), \quad (y,z)\in\Omega_i, \quad i = 0,1,2.$

We introduce the notation:

$$\gamma_i(\lambda) = \sqrt{\lambda^2 - k^2\varepsilon_i}, \quad \lambda\in R, \quad (i = 0,1,2).$$

Fields $u_i(y,z)$ in the domains Ω_i are sought as a Fourier integrals:

$$u_0(y,z) = \int_{-\infty}^{\infty}\frac{C_0^-(\lambda)}{\gamma_0(\lambda)}\cdot Z_0^-(\lambda,z)\cdot e^{i\lambda y}d\lambda, \tag{4}$$

$$u_i(y,z) = \int_{-\infty}^{\infty}\frac{1}{\gamma_i(\lambda)}\left(C_1^+(\lambda)\cdot Z_i^+(\lambda,z) + C_i^-(\lambda)\cdot Z_1^-(\lambda,z)\right)\cdot e^{i\lambda y}d\lambda, \ (y,z)\in\Omega_i, \ i = 1,2; \tag{5}$$

where

$$Z_0^-(\lambda,z) = -\left(1 + h_0\cdot\gamma_0^{-1}(\lambda)\right)^{-1}\cdot e^{-\gamma_0(\lambda)\cdot(z-d_1)},$$

$$Z_i^+(\lambda,z) = \rho_i^{-1}(\lambda)\left(ch(\gamma_i(\lambda)(z-d_{i+1})) + h_i\cdot\gamma_i^{-1}(\lambda)\cdot sh(\gamma_i(\lambda)(z-d_{i+1}))\right),$$

$$Z_i^-(\lambda,z) = -\rho_i^{-1}(\lambda)\left(ch(\gamma_1(\lambda)(z-d_i)) - h_i\cdot\gamma_i^{-1}(\lambda)\cdot sh(\gamma_1(\lambda)(z-d_i))\right),$$

$$\rho_i(\lambda) = 2h_i\cdot\gamma_i^{-1}(\lambda)\cdot ch(\gamma_i(\lambda)(d_i - d_{i+1})) + \left(1 + (h_i\cdot\gamma_i^{-1}(\lambda))^2\right)\cdot sh(\gamma_i(\lambda)(d_i - d_{i+1})).$$

Functions $Z_i^+(\lambda,z)$ and $Z_i^-(\lambda,z)$ has the following properties:

$$\frac{\partial}{\partial z}Z_i^+(\lambda,d_i) + h_i\cdot Z_i^+(\lambda,d_i) = \gamma_i(\lambda), \quad \frac{\partial}{\partial z}Z_i^+(\lambda,d_{i+1}) - h_i\cdot Z_i^+(\lambda,d_{i+1}) = 0, \quad i = 1,2;$$

$$\frac{\partial}{\partial z}Z_i^-(\lambda,d_i) + h_1\cdot Z_1^-(\lambda,d_i) = 0, \quad \frac{\partial}{\partial z}Z_i^-(\lambda,d_{i+1}) - h_i\cdot Z_i^-(\lambda,d_{i+1}) = \gamma_i(\lambda), \quad i = 0,1,2.$$

The Sommerfeld radiation condition will be satisfied if

$$\mathrm{Re}(\gamma_0(\lambda)) \ge 0, \quad \mathrm{Im}(\gamma_0(\lambda)) \le 0, \quad \lambda\in R.$$

It follows from boundary conditions (1), that $C_2^-(\lambda) = 0$. Introduce the notation

$$f(y)=\frac{\varepsilon_1}{\varepsilon_0}\frac{\partial U}{\partial z}(y,d_1)+h_1\cdot U(y,d_1)\,.$$

Conditions of fields' connection on apertures of screens lead to the equalities:

$$\delta_{0,1}\cdot U(y,d_1)+u_{i-1}(y,d_i)=u_i(y,d_i),\quad y\in L_i,\quad i=1,2; \tag{6}$$

$$f(y)+\frac{\varepsilon_1}{\varepsilon_0}\left(\frac{\partial u_0}{\partial z}(y,d_1)-h_0\cdot u_0(y,d_1)\right)+\left(h_1+\frac{\varepsilon_1}{\varepsilon_0}h_0\right)\cdot u_0(y,d_1)=$$

$$=\frac{\partial u_1}{\partial z}(y,d_1)+h_1\cdot u_1(y,d_1),\quad y\in L_1; \tag{7}$$

$$\frac{\varepsilon_1}{\varepsilon_2}\left(\frac{\partial u_2}{\partial z}(y,d_2)+h_2\cdot u_2(y,d_2)\right)-\left(h_1+\frac{\varepsilon_1}{\varepsilon_2}h_2\right)\cdot u_2(y,d_2)=$$

$$=\frac{\partial u_1}{\partial z}(y,d_2)-h_1\cdot u_1(y,d_2),\qquad y\in L_2. \tag{8}$$

From equations (6)–(8) and field representations (4), (5), we obtain the integral relations:

$$U(y,d_1)+\int_{-\infty}^{\infty}\frac{C_0(\lambda)\cdot Z_0^-(\lambda,d_1)}{\gamma_0(\lambda)}e^{i\lambda y}d\lambda=$$

$$=\int_{-\infty}^{\infty}\frac{1}{\gamma_1(\lambda)}\left(C_1^+(\lambda)\cdot Z_1^+(\lambda,d_1)+C_1^-(\lambda)\cdot Z_1^-(\lambda,d_1)\right)\cdot e^{i\lambda y}d\lambda,\quad y\in L_1; \tag{9}$$

$$\int_{-\infty}^{\infty}\frac{C_2(\lambda)\cdot Z_2^+(\lambda,d_2)}{\gamma_2(\lambda)}\cdot e^{i\lambda y}d\lambda=$$

$$=\int_{-\infty}^{\infty}\frac{1}{\gamma_1(\lambda)}\left(C_1^+(\lambda)\cdot Z_1^+(\lambda,d_2)+C_1^-(\lambda)\cdot Z_1^-(\lambda,d_2)\right)\cdot e^{i\lambda y}d\lambda,\quad y\in L_2; \tag{10}$$

$$\int_{-\infty}^{\infty}C_1^+(\lambda)\cdot e^{i\lambda y}d\lambda-\frac{\varepsilon_1}{\varepsilon_0}\int_{-\infty}^{\infty}C_0^-(\lambda)e^{i\lambda y}d\lambda-$$

$$-2h_1\cdot\int_{-\infty}^{\infty}\frac{C_0^-(\lambda)}{\gamma_0(\lambda)}\cdot Z_0^-(\lambda,d_1)\cdot e^{i\lambda y}d\lambda=f(y),\quad y\in L_1; \tag{11}$$

$$\int_{-\infty}^{\infty}C_1^-(\lambda)\cdot e^{i\lambda y}d\lambda-\frac{\varepsilon_1}{\varepsilon_2}\int_{-\infty}^{\infty}C_2^+(\lambda)e^{i\lambda y}d\lambda+$$

$$+2h_1\cdot\int_{-\infty}^{\infty}\frac{C_2^+(\lambda)}{\gamma_2(\lambda)}\cdot Z_2^+(\lambda,d_1)e^{i\lambda y}d\lambda=0,\quad y\in L_2. \tag{12}$$

Let us introduce the following notation

$$W_0(\lambda)=-\frac{Z_0^-(\lambda,d_1)+1}{\gamma_0(\lambda)},\quad W_1(\lambda)=\frac{Z_1^+(\lambda,d_1)-1}{\gamma_1(\lambda)}=-\frac{Z_1^-(\lambda,d_2)+1}{\gamma_1(\lambda)}, \tag{13}$$

$$W_2(\lambda)=\frac{Z_2^+(\lambda,d_2)-1}{\gamma_2(\lambda)},\quad W_3(\lambda)=\frac{Z_1^+(\lambda,d_2)}{\gamma_1(\lambda)}=-\frac{Z_1^-(\lambda,d_1)}{\gamma_1(\lambda)}=\frac{1}{\gamma_1(\lambda)\cdot\rho(\lambda)}. \tag{14}$$

It is useful to remark that

$$W_i(\lambda)=O\left(\lambda^{-2}\right),\quad\lambda\to\infty,\quad i=0,1,2;\quad W_3(\lambda)=O\left(e^{-\gamma_1(\lambda)(d_1-d_2)}\right).$$

It follows from (9)–(14)

$$\int_{-\infty}^{\infty}\frac{C_0^-(\lambda)}{\gamma_0(\lambda)}e^{i\lambda y}d\lambda+\int_{-\infty}^{\infty}C_0^-(\lambda)\cdot W_0(\lambda)e^{i\lambda y}d\lambda+\int_{-\infty}^{\infty}\frac{C_1^+(\lambda)}{\gamma_1(\lambda)}e^{i\lambda y}d\lambda+$$

$$+\int_{-\infty}^{\infty}C_1^+(\lambda)\cdot W_1(\lambda)e^{i\lambda y}d\lambda-\int_{-\infty}^{\infty}\frac{C_1^-(\lambda)}{\gamma_1(\lambda)\rho(\lambda)}e^{i\lambda y}d\lambda=U(y,d_1),\quad y\in L_1;$$

$$\int_{-\infty}^{\infty}\frac{C_2^+(\lambda)}{\gamma_2(\lambda)}e^{i\lambda y}d\lambda+\int_{-\infty}^{\infty}C_2^+(\lambda)\cdot W_2(\lambda)e^{i\lambda y}d\lambda+\int_{-\infty}^{\infty}\frac{C_1^-(\lambda)}{\gamma_1(\lambda)}e^{i\lambda y}d\lambda+$$

$$+\int_{-\infty}^{\infty}C_1^-(\lambda)\cdot W_1(\lambda)e^{i\lambda y}d\lambda-\int_{-\infty}^{\infty}\frac{C_1^+(\lambda)}{\gamma_1(\lambda)\rho(\lambda)}e^{i\lambda y}d\lambda=0,\quad y\in L_2;\tag{15}$$

$$\int_{-\infty}^{\infty}C_1^+(\lambda)\cdot e^{i\lambda y}d\lambda-\frac{\varepsilon_1}{\varepsilon_0}\int_{-\infty}^{\infty}C_0^-(\lambda)e^{i\lambda y}d\lambda+2h_1\int_{-\infty}^{\infty}\frac{C_0(\lambda)}{\gamma_0(\lambda)}e^{i\lambda y}d\lambda+$$

$$+2h_1\cdot\int_{-\infty}^{\infty}C_0(\lambda)\cdot W_0(\lambda)e^{i\lambda y}d\lambda=f(y),\quad y\in L_1;\tag{16}$$

$$\int_{-\infty}^{\infty}C_1^-(\lambda)\cdot e^{i\lambda y}d\lambda-\frac{\varepsilon_1}{\varepsilon_2}\int_{-\infty}^{\infty}C_2^+(\lambda)\cdot e^{i\lambda y}d\lambda+2h_1\cdot\int_{-\infty}^{\infty}\frac{C_2^+(\lambda)}{\gamma_2(\lambda)}\cdot e^{i\lambda y}d\lambda+$$

$$+2h_1\cdot\int_{-\infty}^{\infty}C_2^+(\lambda)\cdot W_2(\lambda)e^{i\lambda y}d\lambda=0,\quad y\in L_2.\tag{17}$$

Let us introduce the functions

$$F_i^-(y)=\frac{\partial u_i}{\partial z}(y,d_{i+1})-h_i\cdot u_i(y,d_{i+1})=\int_{-\infty}^{\infty}C_i^-(\lambda)\cdot e^{i\lambda y}d\lambda,\quad y\in R,\quad i=0,1;\tag{18}$$

$$F_i^+(y)=\frac{\partial u_i}{\partial z}(y,d_i)+h_i\cdot u_i(y,d_i)=\int_{-\infty}^{\infty}C_i^+(\lambda)\cdot e^{i\lambda y}d\lambda,\quad y\in R,\quad i=1,2.\tag{19}$$

It follows from (2), (3), (19), (20)

$$F_i^-(y)=0,\quad y\in L_{i+1},\quad i=0,1;\quad F_i^+(y)=0,\quad y\in L_i,\quad i=1,2;\tag{20}$$

$$C_i^+(\lambda)=\frac{1}{2\pi}\int_{L_i}F_i^+(t)\cdot e^{-i\lambda t}dt,\quad i=1,2;\qquad \lambda\in R;$$

$$C_i^-(\lambda)=\frac{1}{2\pi}\int_{L_{i+1}}F_i^-(t)\cdot e^{-i\lambda t}dt,\quad i=0,1;\quad \lambda\in R.$$

With the help of well-known integral representations of Bessel and Neumann functions of zero order

$$J_0(y)=\frac{2}{\pi}\int_0^1\frac{\cos(yt)dt}{\sqrt{1-t^2}},\qquad Y_0(|y|)=-\frac{2}{\pi}\int_0^{\infty}\frac{\cos(yt)dt}{\sqrt{1-t^2}}$$

the relations

$$\int_{-\infty}^{\infty}\frac{C_i^-(\lambda)}{\gamma_i(\lambda)}e^{i\lambda y}d\lambda=\frac{i}{2}\int_{L_{i+1}}H_0^1\left(k\sqrt{\varepsilon_i}|y-t|\right)F_i^-(t)\,dt,\quad i=0,1;\tag{21}$$

$$\int_{-\infty}^{\infty}\frac{C_i^+(\lambda)}{\gamma_i(\lambda)}e^{i\lambda y}d\lambda=\frac{i}{2}\int_{L_i}H_0^1\left(k\sqrt{\varepsilon_i}|y-t|\right)F_i^+(t)\,dt,\quad i=1,2\tag{22}$$

had been obtained. Introduce the notation:

$$Q_i(y,t)=\int_0^\infty W_i(\lambda)\cos(\lambda(y-t))d\lambda, \quad (i=0,\ldots,3). \tag{23}$$

Note that

$$\int_{-\infty}^{\infty} C_i^-(\lambda)\cdot W_j(\lambda)\cdot e^{i\lambda y}d\lambda=\frac{1}{\pi}\int_{L_{i+1}} Q_j(y,t)F_i^-(t)\,dt, \quad i=0,1; \quad j=0,\ldots,3; \tag{24}$$

$$\int_{-\infty}^{\infty} C_i^+(\lambda)\cdot W_j(\lambda)\cdot e^{i\lambda y}d\lambda=\frac{1}{\pi}\int_{L_i} Q_j(y,t)F_i^+(t)\,dt, \quad i=1,2; \quad j=0,\ldots,3. \tag{25}$$

Due to the integral relations (21)–(25) the system of integral equations (15)–(18) leads to a system of integral equations for the unknown functions $F_0^-(y), F_1^\pm(y), F_2^+(y)$:

$$\frac{i}{2}\int_{L_1} H_0^1\left(k\sqrt{\varepsilon_0}|y-t|\right)F_0^-(t)\,dt+\frac{1}{\pi}\int_{L_1} Q_0(y,t)F_0^-(t)\,dt+\frac{i}{2}\int_{L_1} H_0^1\left(k\sqrt{\varepsilon_1}|y-t|\right)F_1^+(t)\,dt+$$

$$+\frac{1}{\pi}\int_{L_1} Q_1(y,t)\cdot F_1^+(t)\,dt-\frac{1}{\pi}\int_{L_2} Q_3(y,t)\cdot F_1^-(t)dt=U(y,d_1), \quad y\in L_1; \tag{26}$$

$$\frac{i}{2}\int_{L_2} H_0^1\left(k\sqrt{\varepsilon_2}|y-t|\right)F_2(t)\,dt+\frac{1}{\pi}\int_{L_2} Q_2(y,t)F_2(t)\,dt+\frac{i}{2}\int_{L_2} H_0^1\left(k\sqrt{\varepsilon_1}|y-t|\right)F_1^-(t)\,dt+$$

$$+\frac{1}{\pi}\int_{L_2} Q_1(y,t)F_1^-(t)\,dt-\frac{1}{\pi}\int_{L_1} Q_3(y,t)F_1^+(t)dt=0, \quad y\in L_2; \tag{27}$$

$$F_1^+(y)=\frac{\varepsilon_1}{\varepsilon_0}F_0^-(y)-ih_1\int_{L_1} H_0^1\left(k\sqrt{\varepsilon_0}|y-t|\right)F_0(t)\,dt- \tag{28}$$

$$-\frac{2h_1}{\pi}\int_{L_1} Q_0(y,t)F_0(t)\,dt+f(y), \quad y\in L_1;$$

$$F_1^-(y)=\frac{\varepsilon_1}{\varepsilon_2}F_2(y)-ih_1\int_{L_2} H_0^1\left(k\sqrt{\varepsilon_2}|y-t|\right)F_2(t)\,dt-\frac{2h_1}{\pi}\int_{L_2} Q_2(y,t)F_2(t)\,dt, \quad y\in L_2. \tag{29}$$

Substituting (28), (29) into (26), (27), we obtain:

$$\frac{1}{\pi}\int_{L_1}\ln|y-t|F_0^-(t)\,dt+\frac{1}{\pi}\int_{L_1} R_0(y,t)F_0^-(t)\,dt+\frac{1}{\pi}\int_{L_2} R_1(y,t)\cdot F_2^+(t)dt=p_0(y), \quad y\in L_1;$$

$$\frac{1}{\pi}\int_{L_2}\ln|y-t|F_2^+(t)\,dt+\frac{1}{\pi}\int_{L_2} R_2(y,t)F_2^+(t)\,dt+\frac{1}{\pi}\int_{L_1} R_3(y,t)F_0^-(t)dt=p_1(y), \quad y\in L_2;$$

where

$$R_i(y,t)=-\ln|y-t|+\frac{\varepsilon_i}{\varepsilon_1+\varepsilon_i}Q_i(y,t)-$$

$$-\frac{\varepsilon_i}{\varepsilon_1+\varepsilon_i}\left(\frac{i\pi}{2}H_0^1\left(k\sqrt{\varepsilon_i}|y-t|\right)+\frac{i\pi}{2}\frac{\varepsilon_1}{\varepsilon_i}H_0^1\left(k\sqrt{\varepsilon_1}|y-t|\right)\cdot Q_2(y,t)\right)-$$

$$-\frac{\varepsilon_i}{\varepsilon_1+\varepsilon_i}\left(h_1\frac{\pi}{2}\int_{L_i}H_0^1\left(k\sqrt{\varepsilon_1}\left|y-s\right|\right)\cdot H_0^1\left(k\sqrt{\varepsilon_i}\left|s-t\right|\right)ds-ih_1\int_{L_1}H_0^1\left(k\sqrt{\varepsilon_1}\left|y-s\right|\right)\cdot Q_i(s,t)ds\right)+$$

$$+\frac{\varepsilon_i}{\varepsilon_1+\varepsilon_i}\left(ih_1\int_{L_1}Q_1(y,s)\cdot H_0^1\left(k\sqrt{\varepsilon_i}\left|t-s\right|\right)ds+\frac{2h_1}{\pi}\int_{L_1}Q_1(y,s)\cdot Q_i(s,t)ds\right),\quad i=0,2;$$

$$R_1(y,t)=\frac{\varepsilon_0\varepsilon_1\varepsilon_2^{-1}}{\varepsilon_1+\varepsilon_0}Q_3(y,t)-$$

$$-\frac{\varepsilon_0}{\varepsilon_1+\varepsilon_0}\left(ih_1\int_{L_2}Q_3(y,s)H_0^1\left(k\sqrt{\varepsilon_2}\left|t-s\right|\right)ds+\frac{2h_1}{\pi}\int_{L_2}Q_3(y,s)Q_2(s,t)ds\right);$$

$$R_3(y,t)=\frac{\varepsilon_2\varepsilon_1\varepsilon_0^{-1}}{\varepsilon_1+\varepsilon_2}Q_3(y,t)-$$

$$-\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left(ih_1\int_{L_1}Q_3(y,s)\cdot H_0^1\left(k\sqrt{\varepsilon_0}\left|t-s\right|\right)ds+\frac{2h_1}{\pi}\int_{L_1}Q_3(y,s)\cdot Q_0(s,t)ds\right);$$

$$p_0(y)=\frac{\varepsilon_0}{\varepsilon_1+\varepsilon_0}\left(\frac{i}{2}\int_{L_1}H_0^1\left(k\sqrt{\varepsilon_1}\left|y-t\right|\right)f(t)\,dt+\int_{L_1}Q_1(y,t)\cdot f(t)\,dt-U(y,d_1)\right);$$

$$p_2(y)=-\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}\int_{L_1}Q_3(y,t)\cdot f(t)\,dt.$$

4. CONCLUSIONS AND FURTHER RESEARCH DIRECTIONS

The initial boundary-value problem had been reduced to the system of integral equations of the first kind. These equations have logarithmic singularity in the kernel: The discrete singularities method or the method of mechanical quadrature can be used for the numerical solutions of these equation systems. In the future it is supposed to consider the mathematical model of waves scattering on the grating consisting of arbitrary finite number of screens.

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ РАССЕЯНИЯ ПЛОСКИХ ВОЛН НА МНОГОСЛОЙНЫХ ИМПЕДАНСНЫХ СТРУКТУРАХ

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Предложен способ построения математических моделей дифракции электромагнитных волн на многослойных не идеально проводящих отражающих структурах. Исходные краевые задачи для уравнений Гельмгольца с граничными условиями третьего рода сведены к системе граничных интегральных уравнений первого рода. При выводе интегральных уравнений был применён метод параметрических представлений интегральных операторов. Для численного решения полученных систем интегральных уравнений можно применить вычислительные схемы метода дискретных особенностей или метода механических квадратур.

Ключевые слова: задачи дифракции, многослойные отражающие структуры, краевые задачи, метод параметрических представлений интегральных операторов.

МАТЕМАТИЧНА МОДЕЛЬ РОЗСІЯННЯ ПЛОСКИХ ХВИЛЬ НА БАГАТОШАРОВИХ ІМПЕДАНСНИХ СТРУКТУРАХ

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Запропоновано спосіб побудови математичних моделей дифракції електромагнітних хвиль на багатошарових не ідеально провідних відбиваючих структурах. Вихідні крайові задачі для рівнянь Гельмгольца з граничними умовами третього роду зведені до системи граничних інтегральних рівнянь першого роду. Виводячи інтегральні рівняння, застосували метод параметричного подання інтегральних операторів. Для чисельного розв'язування отриманих систем інтегральних рівнянь можна застосувати обчислювальні схеми методу дискретних особливостей або методу механічних квадратур.

Ключові слова: задачі дифракції, багатошарові відбиваючі структури, крайові задачі, метод параметричних уявлень інтегральних операторів.