ISSN 2078–5097. Вісн. Львів. ун-ту. Сер. прикл. матем. та інф. 2016. Вип. 24. С. 53–59
Visnyk of the Lviv University. Series Appl. Math. and Informatics. Issue 24. P. 53–59

УДК 519.21:62

THE ASYMPTOTIC DISSIPATIVITY PROPERTY OF THE EVOLUTIONAL PROCESS WITH MARKOV SWITCHING

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The evolutional system in the asymptotic approximation scheme with Markov switching is considered. Sufficient conditions of dissipativity were obtained for the limited evolution of initial process. Under the balance conditions and smoothness conditions of the Lyapunov function of the limited system, asymptotic dissipativity of the initial system was received.

Ключові слова: property of asymptotic dissipativity, diffusion process, singular perturbation problem, Markov switching.

1. INTRODUCTION

Property of dissipativity is widely discussed in the literature. Dissipativity of deterministic and random systems was considered in works of Samoilenko A.M. and Stanzhytskyi O.M. [2], Mazurov O. Yu. [9], Brogliato B. [5] and others. In particular, dissipativity of stochastic systems with random perturbation was set in Hasminskyi R.Z. work [3].

On the other hand, properties of the random evolution in the asymptotic approximation scheme were considered in the works of Koroliuk V.S. [7], where solution of singular perturbation problem was used for establishing form of the limited evolution generator.

2. MAIN RESULT

Consider an evolutional process defined by solution of stochastic differential equation [1]

$$du^{\varepsilon}(t) = C\left(u^{\varepsilon}(t), x\left(\frac{t}{\varepsilon^2}\right)\right)dt + \varepsilon^{-1}C_0\left(u^{\varepsilon}(t), x\left(\frac{t}{\varepsilon^2}\right)\right)dt, \tag{1}$$

where $u(t) \in R^d$ – random evolution, $t \geq 0$; $C_0(u,x) \in C^2(R^d)$ – singular perturbation of regression function $C(u,x), \in C^2(R^d)$; $x(t)$ – Markov process in phase space of states $(X, \boldsymbol{X})$ with stationary distribution $\pi(B), B \in \boldsymbol{X}$[4,8].

Generator of the Markov process $x(t)$ is given by relation

$$\boldsymbol{Q}\varphi(x) = q(x)\int_X Q(x,dy)[\varphi(y) - \varphi(x)], \varphi \in \mathfrak{B}, \tag{2}$$

where $\mathfrak{B}$ – Banach space of real bounded functions $\varphi(x)$ with supremum norm.

For the generator $\boldsymbol{Q}$ potential $\boldsymbol{R}_0 = \Pi - (\Pi + \boldsymbol{Q})^{-1}$ is determined, where $\Pi\varphi(x) = \int_X \pi(dy)\varphi(y)$ – projector on the space of zeroes of the operator $\boldsymbol{Q}$

$$N_{\boldsymbol{Q}} = \{\varphi : \boldsymbol{Q}\varphi = 0\}.$$

Let balance condition holds

$$\Pi\boldsymbol{C}_0(x) \equiv 0, \tag{3}$$

Limited evolution[6] for the system (1) is determined by solution of equation

$$du(t) = a(u)dt + \sigma(u)dw(t), \tag{4}$$

where

$$a(u) = \int_X C_0(u,x)\boldsymbol{R}_0 C_0^{'}(u,x)\pi(dx) + \int_X C(u,x)\pi(dx). \tag{5}$$

For limited diffusion $\sigma(u)$ relation

$$\sigma(u)\sigma^*(u) = B(u)$$

takes place, where

$$B(u) = 2\int_X C_0(u,x)\boldsymbol{R}_0 C_0(u,x)\pi(dx). \tag{6}$$

Limited generator has a form

$$\boldsymbol{L}(u) = a(u)\varphi'(u) + \frac{1}{2}B(u)\varphi''(u).$$

Let operator $\tilde{\boldsymbol{L}}(x)$ has a representation

$$\tilde{\boldsymbol{L}}(x)\varphi(u) = \boldsymbol{L}\varphi(u) - \left[\boldsymbol{C}_0(x)\boldsymbol{R}_0\boldsymbol{C}_0(x) - \boldsymbol{C}(x)\right]\varphi(u).$$

Definition. System (1) is called asymptotic dissipative if limited evolution (2) is dissipative.

Theorem. Let there exists Lyapunov function $V(u) \in C^3(R^d)$ of determined system

$$\frac{du}{dt} = a(u),$$

which satisfies conditions

$$C1: \left|\boldsymbol{C}_0(x)\boldsymbol{R}_0\tilde{\boldsymbol{L}}(x)V(u)\right| < M_1 V(u), M_1 > 0;$$

$$C2: \left|\boldsymbol{C}(x)\boldsymbol{R}_0\boldsymbol{C}_0(x)V(u)\right| < M_2 V(u), M_2 > 0;$$

$$C3: \left|\boldsymbol{C}(x)\boldsymbol{R}_0\tilde{\boldsymbol{L}}(x)V(u)\right| < M_3 V(u), M_3 > 0.$$

And constraints hold

$$\begin{aligned} &a(u)V'(u) < -c_1 V(u), c_1 > 0 \\ &\sup_{u \in R^d} \|\sigma(u)\| < c_2, c_2 > 0. \end{aligned} \tag{7}$$

Then system (1) is asymptotic dissipative.

3. SOLUTION OF THE SINGULAR PERTURBATION PROBLEM

Lemma 1. Generator of two-component Markov process

$$u_t^\varepsilon = u^\varepsilon(t), x_t^\varepsilon = x\left(\frac{t}{\varepsilon^2}\right), t \geq 0$$

has form

$$\boldsymbol{L}^{\varepsilon}\varphi(u,x) = \varepsilon^{-2}\boldsymbol{Q}\varphi(u,x) + \varepsilon^{-1}\boldsymbol{C}_0(x)\varphi(u,x) + \boldsymbol{C}(x)\varphi(u,x), \tag{8}$$

where

$$\boldsymbol{C}_0(x)\varphi(u) = C_0(u,x)\varphi'(u,x), \tag{9}$$

$$\boldsymbol{C}(x)\varphi(u) = C(u,x)\varphi'(u,x). \tag{10}$$

Proof. Proof of the Lemma reduces to finding the conditional expectation, since

$$\begin{aligned}\boldsymbol{L}^{\varepsilon}\varphi(u,x) &= \lim_{\Delta\to 0}\frac{1}{\Delta}E[\varphi(u^{\varepsilon}_{t+\Delta}, x^{\varepsilon}_{t+\Delta}) - \varphi(u,x) \mid u^{\varepsilon}_t = u, x^{\varepsilon}_t = x] = \\ &= \lim_{\Delta\to 0}\frac{1}{\Delta}E_{u,x}[\varphi(u^{\varepsilon}_{t+\Delta}, x^{\varepsilon}_{t+\Delta})] - \varphi(u,x).\end{aligned} \tag{11}$$

Taking into account, that distribution function θ_x of the time spent in the state x has exponential distribution, namely

$$I(\theta_t > \varepsilon^{-2}\Delta) = e^{-\varepsilon^{-2}q(x)\Delta} = 1 - \varepsilon^{-2}q(x)\Delta + o(\Delta);$$

$$I(\theta_t \le \varepsilon^{-2}\Delta) = 1 - e^{-\varepsilon^{-2}q(x)\Delta} = \varepsilon^{-2}q(x)\Delta + o(\Delta),$$

for the conditional expectation have

$$E_{u,x}[\varphi(u^{\varepsilon}_{t+\Delta}, x^{\varepsilon}_{t+\Delta})] = E_{u,x}[\varphi(u^{\varepsilon}_{t+\Delta}, x^{\varepsilon}_{t+\Delta})][I(\theta_t > \varepsilon^{-2}\Delta) + I(\theta_t \le \varepsilon^{-2}\Delta)] =$$

$$= E_{u,x}[\varphi(u_{t+\Delta}, x)]I(\theta_t > \varepsilon^{-2}\Delta) + E_{u,x}[\varphi(u_{t+\Delta}, x_{t+\Delta})]I(\theta_t \le \varepsilon^{-2}\Delta) =$$

$$= E_{u,x}[\varphi(u_{t+\Delta}, x)](1 - \varepsilon^{-2}q(t)\Delta + o(\Delta)) + E_{u,x}[\varphi(u_{t+\Delta}, x_{t+\Delta})](\varepsilon^{-2}q(t)\Delta + o(\Delta)) =$$

$$= E_{u,x}[\varphi(u_{t+\Delta}, x)] - \varepsilon^{-2}q(t)\Delta E_{u,x}[\varphi(u_{t+\Delta}, x)] + \varepsilon^{-2}q(t)\Delta E_{u,x}[\varphi(u_{t+\Delta}, x_{t+\Delta})] + o(\Delta).$$

From the Taylor decomposition to the second term by the variable u get

$$\varepsilon^{-2}q(x)E_{u,x}[\varphi(u_{t+\Delta}, x)]\Delta = \varepsilon^{-2}q(x)E_{u,x}[\varphi(u,x) + \varphi'(u,x)\Delta u] =$$

$$= \varepsilon^{-2}q(x)[E_{u,x}\varphi(u,x)\Delta + \varepsilon^{-2}q(x)E_{u,x}(\varphi'(u,x)\Delta u)]\Delta + o(\Delta).$$

By the substitution obtained result to the ratio of conditional expectation

$$E_{u,x}[\varphi(u_{t+\Delta}, x_{t+\Delta})] = E_{u,x}[\varphi(u_{t+\Delta}, x)] - \varepsilon^{-2}q(x)E_{u,x}[\varphi(u,x)]\Delta -$$

$$-\varepsilon^{-2}q(x)E_{u,x}[\varphi'(u,x)\Delta u]\Delta + \varepsilon^{-2}q(x)E_{u,x}[\varphi(u_{t+\Delta}, x_{t+\Delta})]\Delta + o(\Delta).$$

Using the decomposition to the Teylor series to term $\varepsilon^{-2}q(x)E_{u,x}[\varphi(u_{t+\Delta}, x_{t+\Delta})]\Delta$, obtain

$$\varepsilon^{-2}q(x)E_{u,x}[\varphi(u_{t+\Delta}, x_{t+\Delta})]\Delta = \varepsilon^{-2}q(x)E_{u,x}[\varphi(u, x_{t+\Delta}) + \varphi'(u, x_{t+\Delta})\Delta u]\Delta + o(\Delta) =$$

$$= \varepsilon^{-2}q(x)E_{u,x}[\varphi(u, x_{t+\Delta})]\Delta + \varepsilon^{-2}q(x)E_{u,x}[\varphi'(u, x_{t+\Delta})\Delta u]\Delta + o(\Delta).$$

From here

$$E_{u,x}[\varphi(u_{t+\Delta}, x_{t+\Delta})] = E_{u,x}[\varphi(u_{t+\Delta}, x)] - \varepsilon^{-2}q(x)E_{u,x}[\varphi(u,x)]\Delta -$$

$$-\varepsilon^{-2}q(x)E_{u,x}[\varphi'(u,x)\Delta u]\Delta + \varepsilon^{-2}q(x)E_{u,x}[\varphi(u, x_{t+\Delta})]\Delta + \varepsilon^{-2}q(x)E_{u,x}[\varphi'(u, x_{t+\Delta})\Delta u]\Delta + o(\Delta).$$

Considering the expression obtained for conditional expectation, generator (11) takes the form

$$L^{\varepsilon}\varphi(u,x) = \lim_{\Delta\to 0}\frac{1}{\Delta}[E_{u,x}[\varphi(u_{t+\Delta}, x)] - \varepsilon^{-2}q(x)\lim_{\Delta\to 0}\frac{1}{\Delta}E_{u,x}[\varphi(u,x)]]\Delta -$$

$$-\varepsilon^{-2}q(x)\lim_{\Delta\to 0}\frac{1}{\Delta}E_{u,x}[\varphi'(u,x)\Delta u]\Delta + \varepsilon^{-2}q(x)\lim_{\Delta\to 0}\frac{1}{\Delta}[E_{u,x}[\varphi(u, x_{t+\Delta})]\Delta +$$

$$+\varepsilon^{-2}q(x)\lim_{\Delta\to 0}\frac{1}{\Delta}[E_{u,x}[\varphi'(u,x_{t+\Delta})\Delta u]\Delta-\varphi(u,x)+\lim_{\Delta\to 0}\frac{1}{\Delta}o(\Delta)=$$

$$=\lim_{\Delta\to 0}\frac{1}{\Delta}[E_{u,x}[\varphi(u_{t+\Delta},x)]-\varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi(u,x)]]-$$

$$-\varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi'(u,x)\Delta u]+\varepsilon^{-2}q(x)\lim_{\Delta\to 0}[E_{u,x}[\varphi(u,x_{t+\Delta})]+$$

$$+\varepsilon^{-2}q(x)\lim_{\Delta\to 0}[E_{u,x}[\varphi'(u,x_{t+\Delta})\Delta u]-\varphi(u,x).$$

For (1) on the interval $[t;t+\Delta]$, receive

$$u_{t+\Delta}=u_t+\int_t^{t+\Delta}C\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds+\varepsilon^{-1}\int_t^{t+\Delta}C_0\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds.$$

Thus,

$$\Delta u=u_{t+\Delta}-u_t=\int_t^{t+\Delta}C\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds+\varepsilon^{-1}\int_t^{t+\Delta}C_0\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds.$$

So,

$$\lim_{\Delta\to 0}[E_{u,x}[\varphi'(u,x_{t+\Delta})\Delta u]-E_{u,x}[\varphi'(u,x)\Delta u]]=\lim_{\Delta\to 0}[E_{u,x}[\varphi'(u,x_{t+\Delta})-\varphi'(u,x)]\Delta u]=$$

$$=\lim_{\Delta\to 0}\left[E_{u,x}\left[\varphi'(u,x_{t+\Delta})-\varphi'(u,x)\right]\right]\times$$

$$\times\left(\int_t^{t+\Delta}C\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds+\varepsilon^{-1}\int_t^{t+\Delta}C_0\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds\right)=0.$$

Herefrom for the generator (11), get

$$\boldsymbol{L}^{\varepsilon}\varphi(u,x)=\lim_{\Delta\to 0}\frac{1}{\Delta}[E_{u,x}[\varphi(u_{t+\Delta t},x)]+$$

$$+\varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi(u,x)]+\varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi(u,x_{t+\Delta t})]-\varphi(u,x).$$

For the first and last terms

$$\lim_{\Delta\to 0}\frac{1}{\Delta}[E_{u,x}[\varphi(u_{t+\Delta t},x)]-\varphi(u,x)=\lim_{\Delta\to 0}\frac{1}{\Delta}[E_{u,x}[\varphi(u+\Delta u,x)-\varphi(u,x)]=$$

$$=\lim_{\Delta\to 0}\frac{1}{\Delta}E_{u,x}\left[\varphi\left(u_t+\int_t^{t+\Delta}C\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds+\varepsilon^{-1}\int_t^{t+\Delta}C_0\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds\right)-\varphi(u,x)\right].$$

From the Teylor decomposition have

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E_{u,x}\left[\varphi\left(u_t+\int_t^{t+\Delta}C\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds+\varepsilon^{-1}\int_t^{t+\Delta}C_0\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds\right)-\varphi(u,x)\right]=$$

$$=\lim_{\Delta\to 0}\frac{1}{\Delta}E_{u,x}\left[\varphi(u,x)+\left(\int_t^{t+\Delta}C\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds+\varepsilon^{-1}\int_t^{t+\Delta}C_0\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds\right)\varphi'(u,x)-\right.$$

$$\left.-\varphi(u,x)\right]=$$

$$=\lim_{\Delta\to 0}\frac{1}{\Delta}E_{u,x}\left(\int_t^{t+\Delta}C\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds+\varepsilon^{-1}\int_t^{t+\Delta}C_0\left(u(s),x\left(\frac{s}{\varepsilon^2}\right)\right)ds\right)\varphi'(u,x)=$$

$$=C(u,x)\varphi'(u,x)+\varepsilon^{-1}C_0(u,x)\varphi'(u,x).$$

Taking into account received results, generator (11) has form

$$\boldsymbol{L}^{\varepsilon}\varphi(u,x) = \varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi(u,x_{t+\Delta})] - \varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi(u,x)] +$$
$$+C(u,x)\varphi'(u,x) + \varepsilon^{-1}C_0(u,x)\varphi'(u,x).$$

From the representation of Markov process generator (2)

$$\varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi(u,x_{t+\Delta})] - \varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi(u,x)] =$$
$$= \varepsilon^{-2}q(x)\lim_{\Delta\to 0}E_{u,x}[\varphi(u,x_{t+\Delta}) - \varphi(u,x)] = \varepsilon^{-2}\lim_{\Delta\to 0}\boldsymbol{Q}\varphi(u,x) = \varepsilon^{-2}\boldsymbol{Q}\varphi(u,x).$$

Finally (11) takes the form

$$\boldsymbol{L}^{\varepsilon}\varphi(u,x) = \varepsilon^{-2}\boldsymbol{Q}\varphi(u,x) + C(u,x)\varphi'(u,x) + \varepsilon^{-1}C_0(u,x)\varphi'(u,x),$$

which coincides with (8) taking into account relations (9) and (10).

Lemma 2. Generator $\boldsymbol{L}^{\varepsilon}$ on the perturbed test-function

$$\varphi^{\varepsilon}(u,x) = \varphi(u) + \varepsilon\varphi_1(u,x) + \varepsilon^2\varphi_2(u,x), \varphi(u) \in C^3(R^d) \quad (12)$$

is defined by the ratio

$$\boldsymbol{L}^{\varepsilon}\varphi^{\varepsilon}(u,x) = \varepsilon^{-2}\boldsymbol{Q}\varphi(u) + \varepsilon^{-1}\boldsymbol{C}^1(u,x) + \boldsymbol{C}^2(u,x) + \varepsilon\theta_1(x), \quad (13)$$

where residual term $\theta_1(x)$ has form

$$\theta_1(x) = \boldsymbol{C}(x)\varphi_1(u,x) + \boldsymbol{C}_0(x)\varphi_2(u,x) + \varepsilon\boldsymbol{C}(x)\varphi_2(u,x) \quad (14)$$

and

$$\boldsymbol{C}^1(u,x) = \boldsymbol{Q}\varphi_1(u,x) + \boldsymbol{C}_0(x)\varphi(u), \quad (15)$$

$$\boldsymbol{C}^2(u,x) = \boldsymbol{Q}\varphi_2(u,x) + \boldsymbol{C}(x)\varphi(u) + \boldsymbol{C}_0(x)\varphi_1(u,x). \quad (16)$$

Proof. From the substitution (12) in the relation for the generator (8)

$$\boldsymbol{L}^{\varepsilon}\varphi^{\varepsilon}(u,x) = \varepsilon^{-2}\boldsymbol{Q}\varphi(u) +$$
$$+\varepsilon^{-1}[\boldsymbol{Q}\varphi_1(u,x) + \boldsymbol{C}_0(x)\varphi(u)] +$$
$$+[\boldsymbol{Q}\varphi_2(u,x) + \boldsymbol{C}(x)\varphi(u) + \boldsymbol{C}_0(x)\varphi_1(u,x)] +$$
$$+\varepsilon[\boldsymbol{C}(x)\varphi_1(u,x) + \boldsymbol{C}_0(x)\varphi_2(u,x) + \varepsilon\boldsymbol{C}(x)\varphi_2(u,x)].$$

Using form of expressions (14)-(16), receive (13).

Lemma 3. Solution of singular perturbation problem for the generator $\boldsymbol{L}^{\varepsilon}$ on the perturbed Lyapunov function

$$V^{\varepsilon}(u,x) = V(u) + \varepsilon V_1(u,x) + \varepsilon^2 V_2(u,x), V(u) \in C^4(R^d) \quad (17)$$

provided (3), has representation

$$\boldsymbol{L}^{\varepsilon}V^{\varepsilon}(u,x) = \boldsymbol{L}V(u) + \varepsilon\theta_2(x)V(u), \quad (18)$$

where $\boldsymbol{L}$ - limited generator, which is determined by the ratio

$$\boldsymbol{L}V(u) = \Pi C(u,x)V'(u) + \Pi C_0(u,x)\boldsymbol{R}_0 C_0(u,x)V''(u), \quad (19)$$

and residual term

$$\theta_2(x)V(u) = \boldsymbol{C}(x)\boldsymbol{R}_0\boldsymbol{C}_0(x)V(u) + \boldsymbol{C}_0(x)\boldsymbol{R}_0\tilde{\boldsymbol{L}}(x)V(u) + \varepsilon\boldsymbol{C}(x)\boldsymbol{R}_0\tilde{\boldsymbol{L}}(x)V(u). \quad (20)$$

Proof. Generator (13) on (17) acts as follows

$$\boldsymbol{L}^{\varepsilon}V^{\varepsilon}(u,x) = \varepsilon^{-2}\boldsymbol{Q}V(u) + \varepsilon^{-1}\boldsymbol{C}^1(u,x) + \boldsymbol{C}^2(u,x) + \varepsilon\theta_1(x).$$

Since $V(u)$ does not depend on x, namely $V(u) \in N_Q$, then $\boldsymbol{Q}V(u) \equiv 0$.

From the singular perturbation problem solvability conditions, respectively, get

$$\boldsymbol{C}^1(u,x) = 0, \quad (21)$$

$$\boldsymbol{C}^2(u,x) = \boldsymbol{L}V(u). \quad (22)$$

First, consider equality (21). Taking into account (15) and (17)

$$\boldsymbol{C}^1(u,x) = \boldsymbol{Q}V_1(u,x) + \boldsymbol{C}_0(x)V(u) = 0;$$

$$\boldsymbol{Q}V_1(u,x) = -\boldsymbol{C}_0(x)V(u).$$

From properties of Markov process generator $\boldsymbol{Q}$, obtain

$$V_1(u,x) = \boldsymbol{R}_0\boldsymbol{C}_0(u,x)V(u). \tag{23}$$

Analogous, from expressions (16) and (17), for condition (22) have

$$\boldsymbol{C}^2(u,x) = \boldsymbol{Q}V_2(u,x) + \boldsymbol{C}(x)V(u) + \boldsymbol{C}_0(x)V_1(u,x) = \boldsymbol{L}V(u).$$

Substituting relation (23) into obtained expression

$$\boldsymbol{Q}V_2(u,x) + \boldsymbol{C}(x)V(u) + \boldsymbol{C}_0(x)\boldsymbol{R}_0\boldsymbol{C}_0(x)V(u) = \boldsymbol{L}V(u);$$

$$\boldsymbol{Q}V_2(u,x) + [\boldsymbol{C}(x) + \boldsymbol{C}_0(x)\boldsymbol{R}_0\boldsymbol{C}_0(x)]V(u) = \boldsymbol{L}V(u).$$

Replace $\boldsymbol{L}(x) = \boldsymbol{C}(x) + \boldsymbol{C}_0(x)\boldsymbol{R}_0\boldsymbol{C}_0(x)$.

Then

$$\boldsymbol{Q}V_2(u,x) + \boldsymbol{L}(x)V(u) = \boldsymbol{L}V(u);$$

$$\boldsymbol{Q}V_2(u,x) = \boldsymbol{L}V(u) - \boldsymbol{L}(x)V(u);$$

$$V_2(u,x) = \boldsymbol{R}_0\tilde{\boldsymbol{L}}(x)V(u), \tag{24}$$

where

$$\tilde{\boldsymbol{L}}(x) = \boldsymbol{L} - \boldsymbol{L}(x).$$

Thus, generator (18) and limited generator (19) are obtained.

Now, from substitution expressions (23) and (24) in representation of the residual term (14)

$$\boldsymbol{C}(x)V_1(u,x) + \boldsymbol{C}_0(x)V_2(u,x) + \varepsilon\boldsymbol{C}(x)V_2(u,x) =$$

$$= \boldsymbol{C}(x)\boldsymbol{R}_0\boldsymbol{C}_0(x)V(u) + \boldsymbol{C}_0(x)\boldsymbol{R}_0\tilde{\boldsymbol{L}}(x)V(u) + \varepsilon\boldsymbol{C}(x)\boldsymbol{R}_0\tilde{\boldsymbol{L}}(x)V(u),$$

which is coinciding with a view of (20).

4. PROOF OF THE THEOREM

Proof of the theorem is based on using Model limited theorem of Korolyuk V.S. [8] and Lemmas 1-3 for obtaining convergence of initial process (1) to limited evolution (4).

Then dissipativity of the system (4) is received, similarly to the theorem given in [1].

5. CONCLUSIONS

In the paper the asymptotic dissipativity of random process with Markov switching in the asymptotic approximation scheme is received. Balance condition is crucial for establishing convergence of the initial system to the limited evolution.

Obtained result allows to consider the Lorenz system of dissipative heat distribution with Markov switching.

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Стаття: надійшла до редколегії 13.05.2015
доопрацьована 03.06.2015
прийнята до друку 17.06.2015

ВЛАСТИВІСТЬ АСИМПТОТИЧНОЇ ДИСИПАТИВНОСТІ ЕВОЛЮЦІЙНОГО ПРОЦЕСУ З МАРКОВСЬКИМ ПЕРЕКЛЮЧЕННЯМ

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Розглянуто еволюційну систему в схемі асимптотичної апроксимації з марковським переключенням. Визначено достатні умови дисипативності граничної еволюції вихідного процесу. В умовах балансу та умовах гладкості функції Ляпунова граничної системи отримано асимптотичну дисипативність вихідної системи.

Key words: властивість асимптотичної дисипативності, дифузійний процес, проблема сингулярного збурення, марковське переключення.