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THE DETERMINATION OF COEFFICIENTS OF THE QUATERNION PARAMETER OF LINEAR REGRESSION

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Recently, multivariate models with complex variables and systems of complex equations have been used in economic research. Specifically, economic and mathematical models are often expressed in terms of complex variables. This approach provides compact modeling of complex processes and describes the dependencies between variables differently, compared to models of real variables. This emerging field, termed "complex economics", provides economists with an innovative analytical instrument. The greater the versatility of this tool, the broader the area of research objectives that can be addressed. These complex-valued functions have very important properties. For example, one of these features is the robustness of their estimates in the presence of multicollinearity.

This paper introduces a novel methodology that aims to expand the econometric researcher's arsenal by incorporating quaternions. The quaternion approach aims to show alternative ways of developing a complex economy compared to those described by complex functions. The paper presents formulas necessary to compute the coefficients of the quaternion regression function and demonstrates their application through an example of finding the coefficients of quaternion regression based on the World Bank statistics.

Key words: quaternions, econometrics, least squares method, linear function.

1. Introduction

In most mathematical models, functions of real variables are used to describe and forecast economic processes at various levels [1, 2]. However, to expand the instrumental base for modeling the relationships of economic variables, the functions of complex variables were used, and this allowed to consider the relationships between indicators by combining them into one complex variable and using it as a dependent variable, in contrast to the functions of real variables, which model the dependence of only one indicator [3-5]. Closer relationships between indicators can be represented using quaternion theory. Therefore, the authors propose a new approach that allows combining twice as many indicators into one quaternion variable of the form

$$x = x_r + x_i i + x_j j + x_k k, \tag{1}$$

where $i^2 = -1$, $j^2 = -1$, $k^2 = -1$ and $ij = k$, $ji = -k$, $jk = i$, $kj = -i$, $ki = j$, $ik = -j$.

Note that the norm of the quaternion x is a non-negative real number $x_r^2 + x_i^2 + x_j^2 + x_k^2$ [6].

The use of a variable in the form of a quaternion of the form (1) as a model connecting four variables into one whole allows, on the one hand, to obtain a more compact notation, and on the other hand, to include in the economic-mathematical model more detailed information about modeled object, and also consider them in a close relationship.

2. Construction of the method

To construct a new linear econometric model with quaternion variables, let's consider a matrix model of the form

$$Y = A + B * X + E, \tag{2}$$

where n - dimensional vectors of quaternions Y and X are economic indicators, and quaternions A and B are the coefficients to be found, E - is an error.

This model (2) in expanded notation has the form:

$$y_{rt} + iy_{it} + jy_{jt} + ky_{kt} = (a_r + ia_i + ja_j + ka_k) + (b_r + ib_i + jb_j + kb_k)\cdot$$
$$\cdot(x_{rt} + ix_{it} + jx_{jt} + kx_{kt}) + (e_{rt} + ie_{it} + je_{jt} + ke_{kt}), \tag{3}$$

where $e_{rt} + ie_{it} + je_{jt} + ke_{kt}$ – n-dimensional error vector, which also has the form of a quaternion, and $t = \overline{1, n}$.

To estimate the parameters of this model based on sample observations, the least squares method is used, in which, similar to the case of real variables, the criterion is the minimization of the norm of the error vector

$$\|E\| = \|e_r + ie_i + je_j + ke_k\| = e_r^2 + e_i^2 + e_j^2 + e_k^2 \to min. \tag{4}$$

This criterion has a fairly transparent interpretation: the smaller the norm of the error of the quaternion variable, the smaller its distance to zero, therefore, we choose the estimates where the sum of error distances to zero is minimal.

This requirement should be met on the entire range of values of t. Therefore, in the general case, the criterion for evaluating these coefficients of the quaternion model is written as follows:

$$\sum_t (y_{rt} - a_r - Re(B * X))^2 + \sum_t (y_{it} - a_i - Im_i(B * X))^2 +$$
$$+\sum_t (y_{jt} - a_j - Im_j(B * X))^2 + \sum_t (y_{kt} - a_k - Im_k(B * X))^2 \to min. \tag{5}$$

Therefore, in order to find the parameters of function with real variables (5), which correspond to its minimum, it is necessary to calculate the derivatives of this function by parameters, equate them to zero, and solve the resulting equations with respect to a_r, a_i, a_j, a_k and b_r,b_i, b_j, b_k.

From the necessary conditions of the extremum we obtain the following system of equations

$$\sum_{t=1}^{n} y_{rt} = na_r + b_r \sum_{t=1}^{n} x_{rt} - b_i \sum_{t=1}^{n} x_{it} - b_j \sum_{t=1}^{n} x_{jt} - b_k \sum_{t=1}^{n} x_{kt}, \tag{6}$$

$$\sum_{t=1}^{n} y_{it} = na_i + b_r \sum_{t=1}^{n} x_{it} + b_i \sum_{t=1}^{n} x_{rt} + b_j \sum_{t=1}^{n} x_{kt} - b_k \sum_{t=1}^{n} x_{jt}, \tag{7}$$

$$\sum_{t=1}^{n} y_{jt} = na_j + b_r \sum_{t=1}^{n} x_{jt} + b_i \sum_{t=1}^{n} x_{kt} + b_j \sum_{t=1}^{n} x_{rt} + b_k \sum_{t=1}^{n} x_{it}, \tag{8}$$

$$\sum_{t=1}^{n} y_{kt} = na_k + b_r \sum_{t=1}^{n} x_{kt} - b_i \sum_{t=1}^{n} x_{jt} - b_j \sum_{t=1}^{n} x_{it} + b_k \sum_{t=1}^{n} x_{rt}, \tag{9}$$

$$\sum_{t=1}^{n} y_{rt}x_{rt} - \sum_{t=1}^{n} y_{it}x_{it} - \sum_{t=1}^{n} y_{jt}x_{jt} - \sum_{t=1}^{n} y_{kt}x_{kt} =$$

$$= a_r \sum_{t=1}^{n} x_{rt} - a_i \sum_{t=1}^{n} x_{it} - a_j \sum_{t=1}^{n} x_{jt} - a_k \sum_{t=1}^{n} x_{kt} + b_r \sum_{t=1}^{n} (x_{rt}^2 - x_{it}^2 - x_{jt}^2 - x_{kt}^2) -$$

$$- 2b_i \sum_{t=1}^{n} x_{rt}x_{it} - 2b_j \sum_{t=1}^{n} x_{rt}x_{jt} - 2b_k \sum_{t=1}^{n} x_{rt}x_{kt}\,, \tag{10}$$

$$\sum_{t=1}^{n} y_{rt}x_{it} + \sum_{t=1}^{n} y_{it}x_{rt} + \sum_{t=1}^{n} y_{jt}x_{kt} - \sum_{t=1}^{n} y_{kt}x_{jt} =$$

$$= a_r \sum_{t=1}^{n} x_{it} + a_i \sum_{t=1}^{n} x_{rt} + a_j \sum_{t=1}^{n} x_{kt} - a_k \sum_{t=1}^{n} x_{jt} + b_i \sum_{t=1}^{n} (x_{rt}^2 - x_{it}^2 - x_{jt}^2 - x_{kt}^2) +$$

$$+ 2b_r \sum_{t=1}^{n} x_{rt}x_{it} + 2b_j \sum_{t=1}^{n} x_{rt}x_{kt} - 2b_k \sum_{t=1}^{n} x_{rt}x_{jt}\,, \tag{11}$$

$$\sum_{t=1}^{n} y_{rt}x_{jt} + \sum_{t=1}^{n} y_{jt}x_{rt} - \sum_{t=1}^{n} y_{it}x_{kt} + \sum_{t=1}^{n} y_{kt}x_{it} =$$

$$= a_r \sum_{t=1}^{n} x_{rt} - a_i \sum_{t=1}^{n} x_{kt} + a_j \sum_{t=1}^{n} x_{rt} + a_k \sum_{t=1}^{n} x_{it} + b_j \sum_{t=1}^{n} (x_{rt}^2 - x_{it}^2 - x_{jt}^2 - x_{kt}^2) +$$

$$+ 2b_r \sum_{t=1}^{n} x_{rt}x_{jt} - 2b_i \sum_{t=1}^{n} x_{rt}x_{kt} + 2b_k \sum_{t=1}^{n} x_{rt}x_{it}\,, \tag{12}$$

$$\sum_{t=1}^{n} y_{rt}x_{kt} + \sum_{t=1}^{n} y_{kt}x_{rt} + \sum_{t=1}^{n} y_{it}x_{jt} - \sum_{t=1}^{n} y_{jt}x_{it} =$$

$$= a_r \sum_{t=1}^{n} x_{kt} + a_i \sum_{t=1}^{n} x_{jt} - a_j \sum_{t=1}^{n} x_{it} + a_k \sum_{t=1}^{n} x_{rt} + b_k \sum_{t=1}^{n} (x_{rt}^2 - x_{it}^2 - x_{jt}^2 - x_{kt}^2) +$$

$$+ 2b_i \sum_{t=1}^{n} x_{rt}x_{jt} - 2b_j \sum_{t=1}^{n} x_{rt}x_{it} + 2b_r \sum_{t=1}^{n} x_{rt}x_{kt}\,. \tag{13}$$

Having solved the system (6-13) with respect to the unknowns a_r, a_i, a_j, a_k and b_r, b_i, b_j, b_k, we obtain the coefficients of the linear model of quaternions.

In particular, when we introduce a designation

$$Y_rX_r = \frac{1}{n}\left(\sum_{t=1}^{n} y_{rt}x_{rt} - \sum_{t=1}^{n} y_{it}x_{it} - \sum_{t=1}^{n} y_{jt}x_{jt} - \sum_{t=1}^{n} y_{kt}x_{kt}\right) -$$

$$-\frac{1}{n^2}\left(\sum_{t=1}^{n} y_{rt}\sum_{t=1}^{n} x_{rt} - \sum_{t=1}^{n} y_{it}\sum_{t=1}^{n} x_{it} - \sum_{t=1}^{n} y_{jt}\sum_{t=1}^{n} x_{jt} - \sum_{t=1}^{n} y_{kt}\sum_{t=1}^{n} x_{kt}\right) =$$

$$= (\overline{y_rx_r} - \overline{y_ix_i} - \overline{y_jx_j} - \overline{y_kx_k}) - (\overline{y}_r\overline{x}_r - \overline{y}_i\overline{x}_i - \overline{y}_j\overline{x}_j - \overline{y}_k\overline{x}_k)\,,$$

$$Y_rX_i = \frac{1}{n}\left(\sum_{t=1}^{n} y_{rt}x_{it} + \sum_{t=1}^{n} y_{it}x_{rt} + \sum_{t=1}^{n} y_{jt}x_{kt} - \sum_{t=1}^{n} y_{kt}x_{jt}\right) -$$

$$-\frac{1}{n^2}\left(\sum_{t=1}^{n} y_{rt}\sum_{t=1}^{n} x_{it} + \sum_{t=1}^{n} y_{it}\sum_{t=1}^{n} x_{rt} + \sum_{t=1}^{n} y_{jt}\sum_{t=1}^{n} x_{kt} - \sum_{t=1}^{n} y_{kt}\sum_{t=1}^{n} x_{jt}\right) =$$

$$= \left(\overline{y_rx_i} + \overline{y_ix_r} + \overline{y_jx_k} - \overline{y_kx_j}\right) - \left(\overline{y}_r\overline{x}_i + \overline{y}_i\overline{x}_r + \overline{y}_j\overline{x}_k - \overline{y}_k\overline{x}_j\right),$$

$$Y_rX_j = \frac{1}{n}\left(\sum_{t=1}^{n} y_{rt}x_{jt} - \sum_{t=1}^{n} y_{it}x_{kt} + \sum_{t=1}^{n} y_{jt}x_{rt} + \sum_{t=1}^{n} y_{kt}x_{it}\right) -$$

$$-\frac{1}{n^2}\left(\sum_{t=1}^{n} y_{rt}\sum_{t=1}^{n} x_{jt} - \sum_{t=1}^{n} y_{it}\sum_{t=1}^{n} x_{kt} + \sum_{t=1}^{n} y_{jt}\sum_{t=1}^{n} x_{rt} + \sum_{t=1}^{n} y_{kt}\sum_{t=1}^{n} x_{it}\right) =$$

$$= \left(\overline{y_rx_j} - \overline{y_ix_k} + \overline{y_jx_r} + \overline{y_kx_i}\right) - \left(\overline{y}_r\overline{x}_j - \overline{y}_i\overline{x}_k + \overline{y}_j\overline{x}_r + \overline{y}_k\overline{x}_i\right),$$

$$Y_rX_k = \frac{1}{n}\left(\sum_{t=1}^{n} y_{rt}x_{kt} + \sum_{t=1}^{n} y_{it}x_{jt} - \sum_{t=1}^{n} y_{jt}x_{it} + \sum_{t=1}^{n} y_{kt}x_{rt}\right) -$$

$$-\frac{1}{n^2}\left(\sum_{t=1}^{n} y_{rt}\sum_{t=1}^{n} x_{kt} + \sum_{t=1}^{n} y_{it}\sum_{t=1}^{n} x_{jt} - \sum_{t=1}^{n} y_{jt}\sum_{t=1}^{n} x_{it} + \sum_{t=1}^{n} y_{kt}\sum_{t=1}^{n} x_{rt}\right) =$$

$$= \left(\overline{y_rx_k} + \overline{y_ix_j} - \overline{y_jx_i} + \overline{y_kx_r}\right) - \left(\overline{y}_r\overline{x}_k + \overline{y}_i\overline{x}_j - \overline{y}_j\overline{x}_i + \overline{y}_k\overline{x}_r\right),$$

$$X_{rr} = \left(\overline{x_r^2} - \overline{x_i^2} - \overline{x_j^2} - \overline{x_k^2}\right) - \left(\overline{x_r}^2 - \overline{x_i}^2 - \overline{x_j}^2 - \overline{x_k}^2\right),$$

$$X_{ri} = \overline{x_rx_i} - \overline{x_r}\cdot\overline{x_i},$$

$$X_{rj} = \overline{x_rx_j} - \overline{x_r}\cdot\overline{x_j},$$

$$X_{rk} = \overline{x_rx_k} - \overline{x_r}\cdot\overline{x_k}.$$

Then the solution of the system has the form:

$$a_r = \overline{y_r} - (b_r\overline{x_r} - b_i\overline{x_i} - b_j\overline{x_j} - b_k\overline{x_k})$$

$$a_i = \overline{y_i} - (b_r\overline{x_i} + b_i\overline{x_r} + b_j\overline{x_k} - b_k\overline{x_j})$$

$$a_j = \overline{y_j} - (b_r\overline{x_l} - b_i\overline{x_k} + b_j\overline{x_r} + b_k\overline{x_i})$$

$$a_k = \overline{y_k} - (b_r\overline{x_k} + b_i\overline{x_j} - b_j\overline{x_i} + b_k\overline{x_r})$$

$$b_r = \Big[\big((Y_rX_rX_{rr} + 2Y_rX_kX_{rk})(X_{rr}^2 + 4X_{rj}^2) + 2(Y_rX_jX_{rr} - 2Y_rX_kX_{ri})\cdot$$

$$\cdot (X_{rj}X_{rr} + 2X_{ri}X_{rk})\big)\big((X_{rr}^2 + 4X_{ri}^2)(X_{rr}^2 + 4X_{rj}^2) - 4((2X_{ri}X_{rj})^2 -$$

$$- (X_{rk}X_{rr})^2)\big) + 2\big((Y_rX_iX_{rr} + 2Y_rX_kX_{rj})(X_{rr}^2 + 4X_{rj}^2) -$$

$$- 2(Y_rX_jX_{rr} - 2Y_rX_kX_{ri})(X_{rk}X_{rr} - 2X_{ri}X_{rj})\big)\big((X_{rr}^2 + 4X_{rj}^2)\cdot$$

$$\cdot (X_{rr}X_{ri} - 2X_{rj}X_{rk}) + 2(X_{rr}X_{rk} + 2X_{rj}X_{ri})(X_{rr}X_{rj} + 2X_{ri}X_{rk})\big)\Big]/$$

$$
\begin{aligned}
&/\Big[\big((X_{rr}^2+4X_{rk}^2)(X_{rr}^2+4X_{rj}^2)-4((2X_{rk}X_{ri})^2-(X_{rj}X_{rr})^2)\big)\cdot\\
&\cdot\big((X_{rr}^2+4X_{ri}^2)(X_{rr}^2+4X_{rj}^2)-4((2X_{ri}X_{rj})^2-(X_{rk}X_{rr})^2)\big)+\\
&+4\big((X_{rr}X_{ri}+2X_{rk}X_{rj})(X_{rr}^2+4X_{rj}^2)-2(X_{rj}X_{rr}-2X_{rk}X_{ri})\cdot\\
&\cdot(X_{rk}X_{rr}-2X_{rj}X_{ri})\big)\big((X_{rr}X_{ri}-2X_{rk}X_{rj})(X_{rr}^2+4X_{rj}^2)+\\
&+2(X_{rk}X_{rr}+2X_{rj}X_{ri})(X_{rj}X_{rr}+2X_{rk}X_{ri})\big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
b_i=&\Big[\big((Y_rX_iX_{rr}+2Y_rX_kX_{rj})(X_{rr}^2+4X_{rj}^2)-2(Y_rX_jX_{rr}-2Y_rX_kX_{ri})\cdot\\
&\cdot(X_{rk}X_{rr}-2X_{ri}X_{rj})\big)\big((X_{rr}^2+4X_{rk}^2)(X_{rr}^2+4X_{rj}^2)-4((2X_{rk}X_{ri})^2-\\
&-(X_{rj}X_{rr})^2)\big)-2\big((Y_rX_rX_{rr}+2Y_rX_kX_{rk})(X_{rr}^2+4X_{rj}^2)+\\
&+2(Y_rX_jX_{rr}-2Y_rX_kX_{ri})(X_{rj}X_{rr}+2X_{ri}X_{rk})\big)\big((X_{rr}^2+4X_{rj}^2)\cdot\\
&\cdot(X_{rr}X_{ri}+2X_{rj}X_{rk})-2(X_{rr}X_{rj}-2X_{rk}X_{ri})(X_{rr}X_{rk}-2X_{ri}X_{rj})\big)\Big]/
\end{aligned}
$$

$$
\begin{aligned}
&/\Big[\big((X_{rr}^2+4X_{rk}^2)(X_{rr}^2+4X_{rj}^2)-4((2X_{rk}X_{ri})^2-(X_{rj}X_{rr})^2)\big)\big((X_{rr}^2+4X_{ri}^2)\cdot\\
&\cdot(X_{rr}^2+4X_{rj}^2)-4((2X_{ri}X_{rj})^2-(X_{rk}X_{rr})^2)\big)+4\big((X_{rr}X_{ri}+2X_{rk}X_{rj})\cdot\\
&\cdot(X_{rr}^2+4X_{rj}^2)-2(X_{rj}X_{rr}-2X_{rk}X_{ri})(X_{rk}X_{rr}-2X_{rj}X_{ri})\big)\big((X_{rr}X_{ri}-\\
&-2X_{rk}X_{rj})(X_{rr}^2+4X_{rj}^2)+2(X_{rk}X_{rr}+2X_{rj}X_{ri})(X_{rj}X_{rr}+2X_{rk}X_{ri})\big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
b_j=&\Big[\big((Y_rX_jX_{rr}-2Y_rX_rX_{rj})(X_{rr}^2+4X_{ri}^2)+2(Y_rX_iX_{rr}-2Y_rX_rX_{ri})\cdot\\
&\cdot(X_{rk}X_{rr}-2X_{ri}X_{rj})\big)\big((X_{rr}^2+4X_{rk}^2)(X_{rr}^2+4X_{ri}^2)+4((2X_{rk}X_{ri})^2-\\
&-(X_{rj}X_{rr})^2)\big)+\big((Y_rX_kX_{rr}-2Y_rX_rX_{rk})(X_{rr}^2+4X_{ri}^2)+\\
&+2(2Y_rX_rX_{ri}-Y_rX_iX_{rr})(X_{rj}X_{rr}+2X_{ri}X_{rk})\big)\big((X_{rr}^2+4X_{ri}^2)\cdot\\
&\cdot(X_{rr}X_{ri}+2X_{rj}X_{rk})-2(X_{rr}X_{rj}-2X_{rk}X_{ri})(X_{rr}X_{rk}-2X_{ri}X_{rj})\big)\Big]/
\end{aligned}
$$

$$
\begin{aligned}
&/\Big[\big((X_{rr}^2+4X_{rk}^2)(X_{rr}^2+4X_{ri}^2)+4((2X_{rk}X_{ri})^2-(X_{rj}X_{rr})^2)\big)\big((X_{rr}^2+4X_{ri}^2)\cdot\\
&\cdot(X_{rr}^2+4X_{rj}^2)-4((2X_{ri}X_{rj})^2-(X_{rk}X_{rr})^2)\big)-2\big((X_{rr}X_{rk}+2X_{ri}X_{rj})\cdot\\
&\cdot(X_{rr}X_{rj}+2X_{ri}X_{rk})-(2X_{rj}X_{rk}-X_{ri}X_{rr})(X_{rr}^2+4X_{ri}^2)\big)\big(2(X_{rr}X_{rj}-\\
&-2X_{rk}X_{ri})(X_{rr}X_{rk}-2X_{rj}X_{ri})-(X_{ri}X_{rr}+2X_{rj}X_{rk})(X_{rr}^2+4X_{ri}^2)\big)\Big]
\end{aligned}
$$

$$b_k = \Big[\big((Y_r X_k X_{rr} - 2Y_r X_r X_{rk})(X_{rr}^2 + 4X_{ri}^2) - 2(Y_r X_i X_{rr} - 2Y_r X_r X_{ri})\cdot$$
$$\cdot (X_{rj} X_{rr} + 2X_{ri} X_{rk})\big)\big((X_{rr}^2 + 4X_{rj}^2)(X_{rr}^2 + 4X_{ri}^2) - 4((2X_{rj} X_{ri})^2 -$$
$$- (X_{rk} X_{rr})^2)\big) + \big((Y_r X_j X_{rr} - 2Y_r X_r X_{rj})(X_{rr}^2 + 4X_{ri}^2) - 2(Y_r X_i X_{rr} -$$
$$- 2Y_r X_r X_{ri})(2X_{ri} X_{rj} - X_{rk} X_{rr})\big)\big(4(X_{rr} X_{rk} + 2X_{ri} X_{rj})\cdot$$
$$\cdot (X_{rr} X_{rj} + 2X_{ri} X_{rk}) - 2(2X_{rk} X_{rj} - X_{rr} X_{ri})(X_{rr}^2 + 4X_{ri})\big)\Big]/$$

$$/\Big[\big((X_{rr}^2 + 4X_{rk}^2)(X_{rr}^2 + 4X_{ri}^2) + 4((2X_{rk} X_{ri})^2 - (X_{rj} X_{rr})^2)\big)\big((X_{rr}^2 + 4X_{ri}^2)\cdot$$
$$\cdot (X_{rr}^2 + 4X_{rj}^2) - 4((2X_{ri} X_{rj})^2 - (X_{rk} X_{rr})^2)\big) - 2\big((X_{rr} X_{rk} + 2X_{ri} X_{rj})\cdot$$
$$\cdot (X_{rr} X_{rj} + 2X_{ri} X_{rk}) - (2X_{rj} X_{rk} - X_{ri} X_{rr})(X_{rr}^2 + 4X_{ri}^2)\big)\big(2(X_{rr} X_{rj} -$$
$$- 2X_{rk} X_{ri})(X_{rr} X_{rk} - 2X_{rj} X_{ri}) - (X_{ri} X_{rr} + 2X_{rj} X_{rk})(X_{rr}^2 + 4X_{ri}^2)\big)\Big]$$

It should be noted that such approach to building a model should be used when economic indicators, that are combined into one quaternion variable, are characteristics of one process or phenomenon, i.e. they reflect different aspects of this phenomenon. That is, there must be a relationship both between the variables and between the real and imaginary parts of each variable. The resulting model can be used for multivariate predictive calculations of different indicators.

Example 1. *Consider an example of use of the proposed method with quaternion coefficients to construct a linear regression between the data given in the table.*

Table 1

Statistical data are taken from the World Data Bank [7]

t	y_{rt}	y_{it}	y_{jt}	y_{kt}	x_{rt}	x_{it}	x_{jt}	x_{kt}
2018	32,135	37,017	17,573	12,750	0,471	2,298	13,418	33,408
2019	31,865	39,702	18,392	13,159	0,433	2,440	13,938	33,057
2020	32,316	34,772	19,443	13,086	0,406	3,288	19,272	37,209
2021	30,054	38,065	15,533	18,682	0,390	4,132	21,169	33,261

Applying the formulas (14-21) we obtain the result:

$$a_r = 34,184, \quad a_i = 35,78082, \quad a_j = 31,65692, \quad a_k = 118,5162,$$
$$b_r = -0,16808, \quad b_i = 0,325369, \quad b_j = 0,064241, \quad b_k = 0,012892.$$

So, the trend of the dependence of y_{rt}, y_{it}, y_{jt}, y_{kt} on x_{rt}, x_{it}, x_{jt}, x_{kt} will look like this:

$$y_{rt} + y_{it}i + y_{jt}j + y_{kt}k = 34,184 + 35,78082i + 31,65692j + 118,5162k +$$
$$+(-0,16808 + 0,325369i + 0,064241j + 0,012892k)\cdot(x_{rt} + x_{it}i + x_{jt}j + x_{kt}k).$$

In particular, this formula shows the dependence of spending on education, receipt of grants to Ukraine in general and technical cooperation in particular, and total income in Ukraine on spending on goods and services, spending on research and development, other spending, and total spending.

3. Conclusions

This mathematical model can be used to forecast both economic indicators and any other statistical characteristics. In addition to the analysis of the dynamics of the real and imaginary parts of the quaternion variable, the dynamics of its other characteristics - the modulus and polar angles - may be of interest to researchers. These indicators can complement the analysis of the investigated indicators. The considered method requires additional research, in particular, finding a confidence interval, estimating the error, etc.

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ВИЗНАЧЕННЯ КОЕФІЦІЄНТІВ КВАТЕРНІОННОГО ПАРАМЕТРА ЛІНІЙНОЇ РЕГРЕСІЇ

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Останнім часом у економічних дослідженнях почали використовувати багатофакторні моделі з комплексними змінними та системи комплексних рівнянь. Такий підхід забезпечує компактне моделювання складних процесів та описує залежності між змінними іншим способом порівняно з моделями дійсних змінних. Запропоновано новий підхід, який ще більше розширить інструментальну базу економетричних досліджень на підставі використання кватерніонів. Наведено формули для обчислення коефіцієнтів кватерніонної функції регресії, використано приклад знаходження коефіцієнтів кватерніонної регресії на основі статистичних даних, взятих зі Світового банку.

Ключові слова: кватерніони, економетрика, метод найменших квадратів, лінійна функція.