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SECANT TYPE METHOD WITH APPROXIMATION OF THE INVERSE OPERATOR FOR THE NONLINEAR LEAST SQUARE PROBLEM

S. Shakhno[1], M. Havdiak[1], R. Iakymchuk[2,3], H. Yarmola[1]

[1]*Ivan Franko National University of Lviv,*
Universytetska str., 1, Lviv, 79000, Ukraine,
[2]*Sorbonne Université LIP6, 4 place Jussieu, 75252 Paris, France,*
[3]*Umeå University, Linnaeus väg 49, 907 36 Umeå, Sweden,*
e-mail: [1]*{stepan.shakhno, mykhaylo.havdiak, halyna.yarmola}@lnu.edu.ua,*
[2]*roman.iakymchuk@sorbonne-universite.fr,* [3]*roman.iakymchuk@umu.se*

In this article, we propose a difference method with successive approximation of the inverse operator for finding an approximate solution of the nonlinear least squares problem. Classical methods are effective for such problems, but still there are types of problems for which they cannot be applied. Besides, these methods require calculation of the inverse matrix or solving a system of linear equations at each iteration, which complicates the task. Hence, the considered method consists of two parts: finding approximation to the solution and to the inverse operator. The first part uses the first-order divided difference of the function instead of Jacobian. The analysis of local convergence of the proposed method is carried out under the classical Lipschitz conditions. This method was also applied for solving test problems, especially with nondifferentiable parts, to show its effectiveness and properties. For comparison, the number of iterations for the Secant method and the method with the approximation of the inverse operator for different initial approximations is shown. Finally, the proposed method can be used for regression analysis problems and in the study of some physical processes if there are difficulties with calculating the derivatives of a nonlinear function and with finding the inverse operator of the divided difference.

Key words: Nonlinear least squares problem, approximation of the inverse operator, difference methods, local convergence, Lipschitz conditions.

1. Introduction

In this article, we propose a method for finding the solution of the nonlinear least squares problem [1, 2]

$$\min_{x\in R^p} f(x) := \frac{1}{2}F(x)^T F(x), \tag{1}$$

where residual function $F : D \subseteq R^p \to R^m$ $(m \geq p)$ is continuously differentiable and nonlinear on x. Denote by $F_i(x)$ an i-th component of the function $F(x)$. The problem is to determine $x_* \in D$ for which

$$f(x_*) = \min_{x\in R^p} f(x).$$

Such problems often appear during the investigation of physical processes or in statistical analysis.

This problem is well studied for general cases. One of the effective methods is the Gauss-Newton method

$$x_{k+1} = x_k - (F'(x_k)^T F'(x_k))^{-1} F'(x_k)^T F(x_k),\ k = 0, 1, \ldots, \tag{2}$$

which was proposed by Friedrich Gauss in 1809 [2]. It is a modification of Newton's method and does not require the calculation of the second-order derivative. However, in some cases, due to the peculiarities of the nonlinear functions (for example, F can be nondifferentiable), it is sometimes impossible to use the Gauss-Newton method for solving problem (1).

The main goal of this article is to provide a convergence analysis of the iterative method for solving the problem (1), which using an approximation of the Fréchet derivative and an approximation of the inverse operator. We investigate the convergence of the proposed method under the classical Lipschitz conditions. Numerical experiments using the test problems are also presented and a comparison of the results against the Secant method for solving the nonlinear least squares problem [3, 5, 7] is conducted.

2. Convergence of the Secant type method with successive approximation of the inverse operator

To find an approximate solution of the problem (1), the following modification of the Gauss-Newton method was proposed and studied in [5, 7]

$$x_{k+1} = x_k - (F(x_k, x_{k-1})^T F(x_k, x_{k-1}))^{-1} F(x_k, x_{k-1})^T F(x_k),\ k = 0, 1, \dots . \tag{3}$$

Here, $x_{-1}, x_0 \in D$ are given initial approximations; $F(x_k, x_{k-1})$ is the divided difference of the first order of the function $F(x)$ at the points x_k, x_{k-1}.

The Gauss-Newton method (2) and the Secant method (3) for solving the nonlinear least-squares problem require the calculation of the inverse matrix or solving system of linear equations, which is not always easy to compute. For such cases, we suggest to find an approximation to the inverse operator using iterative methods.

The inverse operator A^{-1} of a linear operator A can be approximated by Newton's method [11]:

$$A_{k+1} = A_k(2E - AA_k),\ k = 0, 1, \dots, \tag{4}$$

where E is the identity operator and A_0 is initial approximation to A^{-1}. The same publication describes Newton's method with successive approximation of the inverse operator for solving a nonlinear equation

$$\begin{aligned} x_{k+1} &= x_k - A_k F(x_k), \\ A_{k+1} &= A_k(2E - F'(x_{k+1})A_k),\ k = 0, 1, \dots . \end{aligned} \tag{5}$$

Here, A_0 is an initial approximation to the inverse operator $(F'(x_*))^{-1}$; x_0 is an initial approximation to the exact solution of the equation $F(x) = 0$.

The method with successive approximation of the inverse operator consists of two main parts: the first one is for finding an approximation to the problem's solution and the second part is for approximating the inverse operator.

In [8], we propose the Secant method with approximation of the inverse operator for solving (1)

$$\begin{aligned} x_{k+1} &= x_k - A_k B_k^T F(x_k), \\ A_{k+1} &= A_k[2E - B_{k+1}^T B_{k+1} A_k], \quad k = 0, 1, \dots . \end{aligned} \tag{6}$$

Here, $B_k = F(x_k, x_{k-1})$; x_{-1}, x_0 are given initial approximations to x_*; A_0 is an initial approximation to $(F'(x_*)^T F'(x_*))^{-1}$, for example $A_0 = (B_0^T B_0)^{-1}$; E is an identity matrix. The case of $B_k = F'(x_k)$ is described in the article [3].

We will conduct an investigation of the local convergence of the method (6) for problems with zero residual under the classical Lipschitz conditions for the first-order divided differences.

Thus, the condition for the divided difference operator $F(x,y)$

$$||F(x,y) - F(u,v)|| \le L(||x-u|| + ||y-v||) \quad \forall x,y,u,v \in D,$$

is called the Lipschitz condition in the domain D with the Lipschitz constant L. Let $U(x_0,r) = \{x : ||x - x_0)|| < r\}$ be a ball of radius r with center at x_0. Then,

1. the condition $||F(x,y) - F'(x_0)|| \le L(||x-x_0|| + ||y-x_0||) \quad \forall x,y \in U(x_0,r)$ is called the center Lipschitz condition in the ball $U(x_0,r)$ with the constant L;
2. the condition $||F'(x_0)^{-1}F(x,x_0) - I|| \le L||x-x_0|| \quad \forall x \in U(x_0,r)$ is called the radius Lipschitz condition in the ball $U(x_0,r)$ with the constant L.

The conditions described above are called the classical Lipschitz conditions [9].

Theorem 1. *Let F be a nonlinear operator defined on an open convex set D of a Banach space R^p with values in a Banach space R^m. Assume that:*

1) a problem (1) has a solution $x_* \in D$, such that $F(x_*) = 0$, an operator $A_* = [F'(x_*)^T F'(x_*)]^{-1}$ exists and

$$\|A_*\| \le B; \tag{7}$$

2) in the closed ball $\overline{U(x_0,r)} = \{x : \|x - x_*\| \le r_0\}$, where

$$r_0 = \max\{\|x_0 - x_*\|,\ \|x_{-1} - x_*\|,\ \|A_0 - A_*\|\},$$

the following conditions are satisfied:

$$\max\{\|F'(x_*)\|,\ \|F'(x_*)^T\|\} \le C, \tag{8}$$

$$\|F'(x_*) - F(x,y)\| \le L(\|x - x_*\| + \|y - x_*\|); \tag{9}$$

3) initial approximations x_{-1}, x_0 and A_0 are such that

$$q < 1, \tag{10}$$

where

$$q = \max\{a_1 r_0,\ a_2\}, \quad a_1 = C^2 + (B + r_0)(3CL + 2L^2 r_0),$$

and

$$a_2 = C^2 r_0 + (B + r_0)^2 [4CL + 4L^2 r_0].$$

Then, sequences $\{x_k\}$ and $\{A_k\}$ converge to x_* and A_*, respectively. Moreover, the following estimates are fulfilled

$$\|x_k - x_*\| \le q^{c_k} r_0, \quad \|A_k - A_*\| \le q^{g_k} r_0, \tag{11}$$

where

$$c_{-1} = -1,\ c_0 = 0,\ c_k = c_{k-2} + c_{k-1},\ k = 1,2,\ldots,\ g_k = c_{k-1} + 1,\ k = 0,1,\ldots.$$

Proof. The proof is performed by mathematical induction. It follows from

$$\|x_0 - x_*\| \le r_0 = q^{c_0} r_0, \quad \|A_0 - A_*\| \le r_0 = q^{g_0} r_0,$$

so $x_0 \in \overline{U(x_0,r)}$ and (11) is true for $k=0$. Suppose that $x_k \in \overline{U(x_0,r)}$ and the estimate (11) is true for $k \geq 0$. Because of $q<1$, then

$$r_k = \max\{\|x_k - x_*\|, \|A_k - A_*\|\} \leq r_0.$$

Considering (7) and the definition of r_0, we get

$$\|A_k\| \leq \|A_*\| + \|A_k - A_*\| \leq B + r_k \leq B + r_0. \tag{12}$$

We obtain from the first equality of (6) and Taylor's formula

$$\begin{aligned} x_* - x_{k+1} &= x_* - x_k + A_k B_k^T (F(x_k) - F(x_*)) = \\ &= x_* - x_k + A_k B_k^T F(x_k, x_*)(x_k - x_*) = \\ &= [E - A_k B_k^T F(x_k, x_*)](x_* - x_k). \end{aligned} \tag{13}$$

It follows from

$$\begin{aligned} F'(x_*)^T F'(x_*) - B_k^T F(x_k, x_*) &= F'(x_*)^T F'(x_*) - B_k^T F'(x_*) + \\ &\quad + B_k^T F'(x_*) - B_k^T F(x_k, x_*) = \\ &= [F'(x_*)^T - B_k^T] F'(x_*) + B_k^T [F'(x_*) - F(x_k, x_*)], \end{aligned}$$

and taking into account the conditions of the theorem and the inequality

$$\|B_k^T\| \leq \|F'(x_*)^T\| + \|B_k^T - F'(x_*)^T\| \leq C + L(\|x_* - x_k\| + \|x_* - x_{k-1}\|), \tag{14}$$

we obtain

$$\begin{aligned} \|F'(x_*)^T F'(x_*) - B_k^T F(x_k, x_*)\| &\leq CL(2\|x_* - x_k\| + \|x_* - x_{k-1}\|) + \\ &\quad + L^2(\|x_* - x_k\| + \|x_* - x_{k-1}\|)\|x_* - x_k\|. \end{aligned} \tag{15}$$

Based on conditions (8), (9), the estimates (12), (14), (15) and since

$$\begin{aligned} E - A_k B_k^T F(x_k, x_*) &= A_* F'(x_*)^T F'(x_*) - A_k B_k^T F(x_k, x_*) = \\ &= A_k [F'(x_*)^T F'(x_*) - B_k^T F(x_k, x_*)] + \\ &\quad + (A_* - A_k) F'(x_*)^T F'(x_*), \end{aligned}$$

we have that

$$\begin{aligned} \|E - A_k B_k^T F(x_k, x_*)\| &\leq \|A_k\| \|F'(x_*)^T F'(x_*) - B_k^T F(x_k, x_*)\| + \\ &\quad + \|F'(x_*)^T\| \|F'(x_*)\| \|A_* - A_k\| \leq \\ &\leq C^2 \|A_* - A_k\| + \\ &\quad + (B + \|A_* - A_k\|)[CL(2\|x_* - x_k\| + \|x_* - x_{k-1}\|) + \\ &\quad + L^2(\|x_* - x_k\| + \|x_* - x_{k-1}\|)\|x_* - x_k\|]. \end{aligned} \tag{16}$$

We obtain from (13) and (16) that

$$\begin{aligned}
\|x_* - x_{k+1}\| &\le \|E - A_k B_k^T F(x_k, x_*)\| \|x_* - x_k\| \le \\
&\le \Big[C^2\|A_* - A_k\| + (B + \|A_* - A_k\|)[CL(2\|x_* - x_k\| + \|x_* - x_{k-1}\|) + \\
&\quad + L^2(\|x_* - x_k\| + \|x_* - x_{k-1}\|)\|x_* - x_k\|]\Big] \|x_* - x_k\| \le \\
&\le \Big[C^2\|A_* - A_k\| + (B + \|A_* - A_k\|)(3CL + 2L^2\|x_* - x_k\|) \times \\
&\quad \times \|x_* - x_{k-1}\|\Big] \|x_* - x_k\| \le \\
&\le \Big[C^2\|A_* - A_k\| + (B + r_0)(3CL + 2L^2 r_0)\|x_* - x_{k-1}\|\Big] \|x_* - x_k\| \le \\
&\le \Big[C^2 q^{g_k} r_0 + (B + r_0)(3CL + 2L^2 r_0) q^{c_{k-1}} r_0\Big] q^{c_k} r_0 < \\
&< \Big[C^2 + (B + r_0)(3CL + 2L^2 r_0)\Big] q^{c_k + c_{k-1}} r_0^2 = q^{c_{k+1}} r_0. \qquad (17)
\end{aligned}$$

Thus,

$$\|x_* - x_{k+1}\| \le q^{c_{k+1}} r_0 < r_0$$

and

$$x_{k+1} \in \overline{U(x_0, r)}.$$

We obtain from the second equality (6)

$$A_* - A_{k+1} = (A_* - A_k)F'(x_*)^T F'(x_*)(A_* - A_k) - A_k(F'(x_*)^T F'(x_*) - B_{k+1}^T B_{k+1})A_k.$$

Considering the last assumption, estimates (8), (9), and (17), we attain

$$\begin{aligned}
\|A_* - A_{k+1}\| &\le \|A_k\|^2 \|F'(x_*)^T F'(x_*) - B_{k+1}^T B_{k+1}\| + \\
&\quad + \|F'(x_*)^T\| \|F'(x_*)\| \|A_* - A_k\|^2 \le \\
&\le C^2\|A_* - A_k\|^2 + (B + \|A_* - A_k\|)^2 [2CL(\|x_* - x_k\| + \\
&\quad + \|x_* - x_{k+1}\|) + L^2(\|x_* - x_k\| + \|x_* - x_{k+1}\|)^2] \le \\
&\le C^2 q^{2g_k} r_0^2 + (B + r_0)^2 [4CL q^{c_k} r_0 + 4L^2 q^{2c_k} r_0^2] < \\
&< \Big[C^2 r_0 + (B + r_0)^2 [4CL + 4L^2 r_0]\Big] q^{c_k} r_0 = q^{g_{k+1}} r_0.
\end{aligned}$$

That is, (11) is fulfilled for $k+1$. The convergence of sequences $\{x_k\}$ and $\{A_k\}$ follows from the estimate (11) for $k \to \infty$. □

3. Numerical Results

We apply the considered methods for solving test problems for the cases when $m = p$ and $m \ne p$. The first two problems can not be solved by the Gauss-Newton method, because they contain nondifferentiable parts. We use the condition $||x_{k+1} - x_k|| \le \varepsilon$, where $\varepsilon = 10^{-8}$, for stopping the computational process.

Table 1

Approximations to the solution by the method (3), residuals and divided differences for Example 4

k	x_k	$\|F(x_k)\|$	$F(x_k, x_{k-1})$
0	$(1.0, 1.6)$	3.28665389	$\begin{bmatrix} 1.88878889 & -1.0 \\ 1.0 & 3.31101111 \end{bmatrix}$
1	$(1.26714515, 2.50458079)$	0.82873749	$\begin{bmatrix} 2.37825626 & -1.0 \\ 1.0 & 4.21569191 \end{bmatrix}$
2	$(1.14292999, 2.33992414)$	0.12312023	$\begin{bmatrix} 2.52118625 & -1.0 \\ 1.0 & 4.95561605 \end{bmatrix}$
3	$(1.15847877, 2.36137145)$	0.00350551	$\begin{bmatrix} 2.41251988 & -1.0 \\ 1.0 & 4.81240671 \end{bmatrix}$
4	$(1.15936717, 2.36182509)$	$1.76618586e-05$	$\begin{bmatrix} 2.42895706 & -1.0 \\ 1.0 & 4.83430766 \end{bmatrix}$
5	$(1.15936085, 2.36182434)$	$5.58477895e-09$	$\begin{bmatrix} 2.42983913 & -1.0 \\ 1.0 & 4.83476054 \end{bmatrix}$
6	$(1.15936085, 2.36182434)$	$1.35691205e-14$	$\begin{bmatrix} 2.42983276 & -1 \\ 1 & 4.83476011 \end{bmatrix}$

Calculations were performed using Python 3.9.2 and 2,4 GHz 8-Core Intel Core i9/64 GB.

Example 4. *Consider a system of nonlinear equations*

$$\begin{aligned} x_1^2 - x_2 + 1 + \frac{1}{9}|x_1 - 1| = 0, \\ x_2^2 + x_1 - 7 + \frac{1}{9}|x_2| = 0. \end{aligned} \tag{18}$$

The solution of this problem is $x_* \approx (1.15936717, 2.36182509)$.

For initial approximations, we use $x_0 = (1.0, 1.6)$ and $x_{-1} = (0.9999, 1.5999)$. The Table 1 shows the results obtained by the method (3) at each iteration.

Let us apply the method (6) for the system of equations from Example 4. Table 2 shows the results of the Secant type method with successive approximation of the inverse operator for the problem (18) with the same initial approximations.

Example 5. *Consider a system of nonlinear equations, when $m \neq p$:*

$$\begin{aligned} x_1^2 + 3x_2 - 7 + |2.5 - 2x_1| = 0, \\ 2x_2 e^{x_1+1} - x_2^2 - |\sqrt{-x_1}x_2 + 1.5x_2 - 2| = 0, \\ x_1^2 x_2 - |x_2| = 0. \end{aligned} \tag{19}$$

For this problem $x_* = (-1; 0.5)$.

Table 2

Approximations to the solution by the method (6), residuals and divided differences for Example 4

k	x_k	$\|F(x_k)\|$	$F(x_k, x_{k-1})$
0	$(1.0, 1.6)$	3.28665389	$\begin{bmatrix} 1.88878889 & -1 \\ 1 & 3.31101111 \end{bmatrix}$
1	$(1.26714515, 2.50458080)$	0.82873751	$\begin{bmatrix} 2.37825626 & -1.0 \\ 1.0 & 4.21569191 \end{bmatrix}$
2	$(1.15445344, 2.39294403)$	0.15270233	$\begin{bmatrix} 2.53270971 & -1.0 \\ 1.0 & 5.00863594 \end{bmatrix}$
3	$(1.15861503, 2.36306145)$	0.00605964	$\begin{bmatrix} 2.42417958 & -1.0 \\ 1.0 & 4.86711659 \end{bmatrix}$
4	$(1.15935080, 2.36183880)$	$7.13645916e-05$	$\begin{bmatrix} 2.42907694 & -1.0 \\ 1.0 & 4.83601136 \end{bmatrix}$
5	$(1.15936085, 2.36182435)$	$3.62087881e-08$	$\begin{bmatrix} 2.42982277 & -1.0 \\ 1.0 & 4.83477426 \end{bmatrix}$
6	$(1.15936085, 2.36182434)$	$1.25322626e-13$	$\begin{bmatrix} 2.42981257 & -1 \\ 1 & 4.83475321 \end{bmatrix}$

Table 3 shows results for Example 5 with different initial approximations. Additional starting point is calculated by formula $x_{-1} = x_0 - 0.0001$.

Table 3

The number of iterations and time to obtain an approximation to the solution $x_* = (-1; 0.5)$ by both methods with the accuracy $\varepsilon = 10^{-8}$

Initial approximation	Method (3)		Method (6)	
	Iterations	Time	Iterations	Time
$x_0 = (-0.5, -3.0)$	9	0.00687	11	0.00771
$x_0 = (-0.5, -3.5)$	10	0.00701	12	0.00782
$x_0 = (-2.0, -0.5)$	8	0.00503	9	0.00574
$x_0 = (-2.5, 3.0)$	11	0.00749	12	0.00825
$x_0 = (-2.5, -1.0)$	10	0.00684	11	0.007
$x_0 = (-4.6, 3.6)$	14	0.01116	15	0.00994
$x_0 = (-2.2, 8.2)$	14	0.00966	15	0.00978
$x_0 = (-2.4, 4.0)$	13	0.00851	13	0.00825

For the test problems from [4], we give the number of iterations for calculating the approximate solution using the difference methods (3) and (6). We consider problems with zero and non-zero residuals.

The following test problems were used for numerical calculations [4]:

Table 4

The number of iterations to obtain an approximation to the solutions of the test problems by both methods with the accuracy $\varepsilon = 10^{-8}$

Problem	Method (3)		Method (6)	
	Iterations	Time	Iterations	Time
Rosenbrock function $p=2,\ m=2,\ x_0=(1,10)$ $x_*=(1,1),\ f(x_*)=0$	3	0.00213	3	0.0013
Beale function $p=2,\ m=3,\ x_0=(1.0,-1.5)$ $x_*=(3,0.5),\ f(x_*)=0$	11	0.00696	16	0.00886
Helical valley function $p=3,\ m=3,\ x_0=(1.0,-0.2,-3.0)$ $x_*=(1,0,0),\ f(x_*)=0$	6	0.00555	9	0.00791
Gaussian function $p=3,\ m=15,\ x_0=(-3.0,1.0,-1.0)$ $x_*=(0.4,1,0),\ f=1.12793...10^{-8}$	13	0.1179	14	0.06232
Freudenstein and Roth function $p=2,\ m=2,\ x_0=(10,8)$ $x_*=(5,4),\ f(x_*)=0$	10	0.00547	13	0.00631
Box three-dimensiomal function $p=3,\ m=250,\ x_0=(0.5,9,2)$ $x_*=(1,10,1),\ f(x_*)=0$	10	4.78794	12	3.82081

1. Rosenbrock function

$$F_1(x) = 10(x_2 - x_1^2),$$
$$F_2(x) = 1 - x_1.$$

2. Beale function

$$F_i(x) = y_i - x_1(1 - x_2^i),$$
$$\text{where} \quad y_1 = 1.5, \quad y_2 = 2.25, \quad y_3 = 2.625.$$

3. Helical valley function

$$F_1(x) = 10[x_3 - 10\Theta(x_1, x_2)],$$
$$F_2(x) = 10[(x_1^2 + x_2^2)^{1/2} - 1],$$
$$F_3(x) = x_3,$$

$$\Theta(x_1, x_2) = \begin{cases} \frac{1}{2\pi} \arctan \frac{x_2}{x_1}, & if \quad x_1 > 0, \\ \frac{1}{2\pi} \arctan \frac{x_2}{x_1} + 0.5, & if \quad x_1 < 0. \end{cases}$$

4. Gaussian function

$$F_i(x) = x_1 e^{-x_2(t_i - x_3)^2/2} - y_i, \quad t_i = (8-i)/2.$$

5. Freudenstein and Roth function

$$F_1(x) = -13 + x_1 + ((5 - x_2) - 2)x_2,$$
$$F_2(x) = -29 + x_1 + ((x_2 + 1)x_2 - 14)x_2.$$

6. Box three-dimensional function

$$F_i(x) = e^{-t_i x_1} - e^{-t_i x_2} - x_3\left(e^{-t_i} - e^{-10t_i}\right), \quad t_i = 0.1i.$$

Table 4 shows number of iterations for finding the solution of the described functions by methods (3), (6), where $\varepsilon = 10^{-8}$.

4. Conclusions

This article describes the nonlinear least squares problem and the methods for find an approximation to its solution. The classical Gauss-Newton method and the Secant method require calculations of inverse matrix or solving system of linear equations. Given this, the Secant type method with the successive approximation of the inverse operator is proposed and studied. This method is tested on problems with zero and non-zero residuals, with different numbers of unknowns and equations. The method with the successive approximation of the inverse operator takes more iterations than the basic Secant method, but it does not require to solve the system of linear equations or to calculate the inverse matrix at each iteration. This is the main advantage of the proposed method.

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МЕТОД ТИПУ ХОРД З АПРОКСИМАЦІЄЮ ОБЕРНЕНОГО ОПЕРАТОРА ДЛЯ НЕЛІНІЙНИХ ЗАДАЧ НАЙМЕНШИХ КВАДРАТІВ

С. Шахно[1], М. Гавдяк[1], Р. Якимчук[2,3], Г. Ярмола[1]

[1]*Львівський національний університет імені Івана Франка, вул. Університеська, 1, Львів, 79000,*
[2]*Університет Сорбонна – LIP6, 4 place Jussieu, 75252 Paris, France,*
[3]*Університет Умео, Linnaeus väg 49, 907 36 Umeå, Sweden,*
e-mail: [1]*{stepan.shakhno, mykhaylo.havdiak, halyna.yarmola}@lnu.edu.ua*,
[2]*roman.iakymchuk@sorbonne-universite.fr*, [3]*roman.iakymchuk@umu.se*

Запропоновано різницевий метод з послідовною апроксимацією оберненого оператора для пошуку наближеного розв'язку нелінійної задачі про найменші квадрати. Досліджено його збіжність і проведено числовий експеримент. Класичні методи розв'язування ефективні, проте існують типи задач, для яких їх неможливо застосувати. Також ці методи потребують обчислення оберненої матриці або розв'язування системи лінійних алгебричних рівнянь на кожній ітерації, що може ускладнити пошук наближеного розв'язку. На противагу їм запропонований метод використовує замість якобіана поділену різницю першого порядку від нелінійної функції. Розглянутий метод складається з двох частин: перша частина призначена для знаходження наближень до розв'язку; друга – використовує апроксимацію оберненого оператора замість розв'язування систем лінійних алгебричних рівнянь чи пошуку оберненої матриці. Проведено аналіз локальної збіжності та з'ясовано порядок збіжності запропонованого методу за класичних умов Ліпшиця. Виконано числовий експеримент на тестових задачах для вивчення реальних властивостей методу. Також деякі задачі містили недиференційовну частину. Наведено кількість ітерацій необхідну для обчислення наближеного розв'язку з допомогою методу хорд і методу з апроксимацією оберненого оператора, а також результати при різних початкових наближеннях. Основною перевагою запропонованого методу є те, що його можна застосовувати до задач регресійного аналізу і у дослідженні деяких фізичних процесів у випадку, коли виникають труднощі з обчисленням похідних від нелінійної функції та зі знаходженням оберненого оператора.

Ключові слова: нелінійна задача про найменші квадрати, апроксимація оберненого оператора, поділена різниця, різницеві методи, локальна збіжність, умови Ліпшиця.