UDC 519.21

CONTROL PROBLEM FOR THE IMPULSE PROCESS UNDER STOCHASTIC OPTIMIZATION PROCEDURE AND NONCLASSICAL APPROXIMATION

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A stochastic approximation procedure and a limit generator of the original problem are constructed for a system of stochastic differential equations with Markov switching and impulse perturbation under noclassical approximation conditions with control, which is determined by the condition for the extremum of the quality criterion function. The control problem using the stochastic optimization procedure is a generalization of the control problem with the stochastic approximation procedure, which was studied in previous works of the authors. This generalization is not simple and requires non-trivial approaches to solving the problem. In particular we discuss how the behavior of the boundary process depends on the pre-limiting stochastic evolutionary system in the ergodic Markov environment. The main assumption is the condition for uniform ergodicity of the Markov switching process, that is, the existence of a stationary distribution for the switching process over large time intervals. This allows one to construct explicit algorithms for the analysis of the asymptotic behavior of a controlled process. An important property of the generator of the Markov switching process is that the space in which it is defined splits into the direct sum of its zero-subspace and a subspace of values, followed by the introduction of a projector that acts on the subspace of zeros. For the first time, a model of the control problem for the diffusion transfer process using the stochastic optimization procedure for control problem is proposed. A singular expansion in the small parameter of the generator of the three-component Markov process is obtained, and the problem of a singular perturbation with the representation of the limiting generator of this process is solved

Key words: random evolution, nonclassical approximation conditions, stochastic optimization procedure, Markov process.

1. Introduction

Analyzing the state of the art concerning asymptotic properties of stochastic evolution models reveals that a complete theory is still to be worked out. Well understood are the models which are given by stochastic differential equations with Markov switchings and impulse or continuous-type perturbations in the classical schemes of averaging or diffusion approximation [2]-[4]. Also, the asymptotic behavior was investigated of impulse processes with Markov switchings under the conditions of Levy approximation [1], [8]-[11]. Thus, it seems natural to develop a theory of evolution equations with Markov switchings and random perturbations in nonclassical approximation schemes. Establishing convergence of the stochastic optimization procedure is an important puprose of system analysis in the uncertainties, which can be modeled using an ergodic Markov environment. The relevance of determining new properties and generalizations of optimization algorithms that use random-ness in the process of finding the optimum is evidenced by numerous applications in control theory, information transfer theory, and also in solving nonparametric problems of mathematical statistics. In present article, we focus on the study of the evolutionary system in the form of a perturbed controlled impulse process with Markov switching under the conditions of the existence of a single extremum point of the function for assessing the quality of control. We consider some prelimit evolution models with a small normalization parameter. We find the form of the limit generators for the impulse processes and the control of dynamical system in the schemes of the Levy approximation, and the stochastic optimization. Further, we provide conditions which ensure weak convergence of a controlled evolution model with Markov switching and impulse perturbation (assuming uniqueness of the equilibrium point for the quality criterion for which the stochastic optimization procedure is given). It is important that the asymptotic behavior of the limit process is concluded with the help of the analysis of parameters of the pre-limit system [13].

1.1. Model problem

We investigate the stochastic evolution system in the ergodic Markovian environment given by a stochastic evolutionary equation [7],[8], [10]

$$dy^\varepsilon(t) = C(y^\varepsilon(t), x(t/\varepsilon^2)) + d\eta^\varepsilon(t, u^\varepsilon(t)), \tag{1}$$

where $y^\varepsilon(t)$ is a solution, $x(t)$, $t \geq 0$, is a uniformly ergodic Markov process, that is defined on a standard phase space $(X, \mathbf{X})$ [3] by the generator

$$\mathbf{Q}\varphi(x) = q(x) \int_X P(x, dy)[\varphi(x) - \varphi(y)]$$

on the Banach space $B(x)$ of real valued bounded functions with the supremum norm

$$\|\varphi(x)\| = \sup_{x \in X} |\varphi(x)|$$

$u^\varepsilon(x)$is a control function. A stochastic kernel $P(x, B)$, $x \in X$, $B \in \mathbf{X}$ defines uniformly ergodic embedded Markov chain $x_n = x(\tau_n)$, $n \geq 0$, that has stationary distribution $\rho(B)$, $B \in \mathbf{X}$. Stationary distribution $\pi(B)$, $B \in \mathbf{X}$ for Markov process $x(t)$, $t \geq 0$, is defined by relation [3]

$$\pi(dx)q(x) = q\rho(x), \;\; q = \int_X \pi(dx)q(x).$$

The impulse perturbation process $\eta^\varepsilon(t)$, $t \geq 0$, is given by the relation

$$\eta^\varepsilon(t, u^\varepsilon(t)) = \int_0^t \eta^\varepsilon(s, u^\varepsilon(s), x(s/\varepsilon^2))ds \tag{2}$$

where a set of processes with independent increments $\eta^\varepsilon(t, u, x)$, $t \geq 0$, $x \in X$, $u \in U$, is determined by the generators

$$\Gamma^\varepsilon(x)\varphi(u, w) = \varepsilon^{-2} \int_R \varphi(u, w + v) - \varphi(u, w)\Gamma^\varepsilon(dv, x), \tag{3}$$

where $x \in X$ and the following Levy's approximation conditions are satisfied ([2,3]):

L1. The approximation of averages [4]

$$\int_R v\Gamma^\varepsilon(dv,x) = \varepsilon a_1(x) + \varepsilon^2(a_2(x)+\theta_a(x)),\ \theta_a(x)\to 0,\ \varepsilon\to 0,$$

and

$$\int_R v^2\Gamma^\varepsilon(dv,x) = \varepsilon(b(x)+\theta_b(x)),\ \theta_b(x)\to 0,\ \varepsilon\to 0,$$

L2. The condition imposed on the distribution function

$$\int_R g(v)\Gamma^\varepsilon(dv,x) = \varepsilon^2(\Gamma_g(x)+\theta_g(x)),\ \theta_g(x)\to 0,\ \varepsilon\to 0,$$

for all $g(v)\in C^2(\mathbb{R})$ (the space of real-valued bounded functions such that $g(v)/|v|^2\to 0$, $|v|\to 0$), where measure $\Gamma_g(x)$ is bounded for all $g(v)\in C^2(\mathbb{R})$ and is defined by the relation (functions from the space $C^2(\mathbb{R})$ separate the measures):

$$\Gamma_g(x) = \int_R g(v)\Gamma_0(dv,x),\ g(v)\in C^3(\mathbb{R});$$

L3. The uniform quadratic integrability

$$\sup\lim_{c\to\infty}\int_{|v|>c} v^2\Gamma_0(dv,x) = 0.$$

As an example of a random variable that satisfies the conditions of the Levy approximation[5], we can cite the following α:

$$P\{\alpha = b\} = \varepsilon^2 p,$$

$$P\{\alpha = \varepsilon a_1 + \varepsilon^2 b_1\} = 1-\varepsilon^2 p.$$

Then we have:

$$\mathbf{E}\alpha = \varepsilon a_1 + \varepsilon^2(bp+b_1) + o(\varepsilon^2),$$

$$\mathbf{E}\alpha^2 = \varepsilon^2(b^2p + a_1^2) + o(\varepsilon^2).$$

These moment conditions characterize Levy approximation.

If $a_1 = 0$, we obtain

$$\mathbf{E}\alpha = \varepsilon^2(bp+b_1) + o(\varepsilon^2),$$

$$\mathbf{E}\alpha^2 = \varepsilon^2 b^2 p + o(\varepsilon^2),$$

and assuming $\widetilde{\varepsilon} = \varepsilon^2$ we have moment conditions characterize Poison approximation:

$$\mathbf{E}\alpha = \widetilde{\varepsilon}(bp+b_1) + o(\widetilde{\varepsilon}),$$

$$\mathbf{E}\alpha^2 = \widetilde{\varepsilon} b^2 p + o(\widetilde{\varepsilon}).$$

In general, Poisson approximation conditions have a form

P1. The approximation of averages:

$$b_\varepsilon = \int_{\mathbf{R}} v\widetilde{\Gamma}^\varepsilon(dv) = \varepsilon[b + \theta_b^\varepsilon],$$

and

$$c_\varepsilon = \int_{\mathbf{R}} v^2\widetilde{\Gamma}^\varepsilon(dv) = \varepsilon[c + \theta_c^\varepsilon],$$

where

$$b < +\infty, c < +\infty.$$

P2. Asymptotic representation for intensivity kernel

$$\widetilde{\Gamma}_g^\varepsilon = \int_{\mathbf{R}} g(v)\widetilde{\Gamma}^\varepsilon(dv) = \varepsilon[\widetilde{\Gamma}_g + \theta_g^\varepsilon]$$

for each $g \in C_3(\mathbf{R})$.

$\theta_b^\varepsilon, \theta_c^\varepsilon, \theta_g^\varepsilon$ satisfies condition

$$|\theta_*^\varepsilon| \to 0, \quad \varepsilon \to 0.$$

P3. Absence of diffusion component at limit generator

$$c := \int_{\mathbf{R}} v^2\widetilde{\Gamma}^0(dv).$$

P4. Uniformly quadratic integrability:

$$\lim_{c\to\infty} \int_{|v|>c} v^2\widetilde{\Gamma}^0(dv) = 0.$$

2. Stochastic optimization procedure for control function

We assume that control function $u(t)$[6] for equation (1) defines by quality criterion $G(y, x, u)$, that has unique extremum for each value of process y and for each state x of Markov process $x(t)$ at interval $[\tau_i, \tau_{i+1})$, that is

$$u(t) = u_{x,y}(t) = \arg \mathbf{ext} G(x, y, u(t))$$

for $[\tau_i, \tau_{i+1})$.

We assume also that $G(\cdot, \cdot, u) \in C^2(\mathbb{R}^d)$. Then $u(t)$ is completely determined by system

$$\frac{\partial G(y(t), x(t), u(t))}{\partial u_k} = 0 \ (k = \overline{1, d}) \tag{4}$$

So we consider stochastic optimization procedure [9, 12] for control function $u(t)$

$$du^\varepsilon(t) = \alpha(t)\nabla_{\beta(t)} G(y^\varepsilon(t), x(t/\varepsilon^2), u^\varepsilon(t))dt, \tag{5}$$

where

$$\nabla_{\beta(t)} G(\cdot, \cdot, u) = (G(\cdot, \cdot, u_i^+) - G(\cdot, \cdot, u_i^-))/2\beta(t), \ i = \overline{1, d},$$

$$u_i^\pm = u_i \beta(t) e_i, \ e_i = (0, ..., 1, 0, ..., 0), \ i = \overline{1, d}.$$

General initial conditions has a form

$$y(0) = y_0;\ x(0) = x_0;\ u(0) = u_0. \tag{6}$$

Functions $\alpha(t)$, $\beta(t)$, $t > 0$ are satisfying relations

$$\alpha(t) \to 0,\ \beta(t) \to 0$$

at $t \to \infty$.

3. Main results

Theorem 1. We assume the balance condition

$$\int_X \pi(dx) a_1(x) = 0$$

holds true,

$$C(y, x) \in C(\mathbb{R}^d, \mathbf{X},\ \alpha(t) \to 0,\ \beta(t) \to 0$$

at $t \to \infty$, and quality criterion $G(y, u, x) \in C(\mathbb{R}^d, \mathbf{X}, \mathbb{R}^d)$.

Then weak convergence

$$(y^\varepsilon(t), u^\varepsilon(t), \eta^\varepsilon(t)) \Rightarrow (\hat{y}(t), \hat{u}(t), \hat{\eta}(t)),\ \varepsilon \to 0, \tag{7}$$

holds true, where limit process $(\hat{y}(t), \hat{u}(t), \hat{\eta}(t))$ is defined by generator

$$\mathbf{M}\varphi(y, u, w) = \mathbf{L}\varphi(y, u, w) + \mathbf{B}_t\varphi(y, u, w) \tag{8}$$

where

$$\mathbf{L}\varphi(y, u, w) = \mathbf{C}(y)\varphi(y, u, w) + \mathbf{\Gamma}\varphi(y, u, w),$$

$$\mathbf{C}(y)\varphi(y) = \hat{C}(y)\varphi'(y),$$

$$\hat{C}(y) = \int_X \pi(dy) C(x, y)$$

$$\mathbf{\Gamma}\varphi(u, w) = \hat{a}_2\varphi'_w(u, w) + \frac{1}{2}\sigma^2\varphi''_{ww}(u, w) + \int_R [\varphi(u, w + v) - \varphi(u, w)]\hat{\Gamma}_0(dv),$$

$$\hat{a}_2 = \int_X \pi(dx)(a_2(x) - a_0(x)),$$

$$\sigma^2 = \int_X \pi(dx)(b(x) - b_0(x)) + 2\int_X \pi(dx) a_1(x) R_0 a_1(x),$$

$$a_0(x) = \int_R v\Gamma_0(dv, x),$$

$$b_0(x) = \int_R v^2\Gamma_0(dv, x),$$

$$\hat{\Gamma}_0(v) = \int_X \pi(dx)\Gamma_0(v,x),$$

$$\mathbf{B}_t\varphi(y,u,w) = \alpha(t)\nabla_{\beta(t)}G(y,u)\varphi'_u(y,u,w),$$

$$\nabla_{\beta(t)}G(y,u) = \int_X \nabla_{\beta(t)}G(y,u,x)\pi(dx).$$

Consequence 1. Limit process $(\hat{y}(t),\hat{u}(t))$ for control problem (5), (6), (7), is determined by stochastic differential equation

$$d\hat{y}(t) = a(\hat{y}(t))dt + d\eta((\hat{y}(t),\hat{u}(t))), \tag{9}$$

$$d\hat{u}(t) = \alpha(t)\nabla_{\beta(t)}G(\hat{y}(t),\hat{u}(t))dt, \tag{10}$$

under initial conditions (7).

Consequence 2. Assuming that impulse perturbation process $y(t)$, defined at series scheme by stochastic differential equation

$$dy^\varepsilon(t) = C(y^\varepsilon(t), x(t/\varepsilon^2), u^\varepsilon(t))dt + d\eta^\varepsilon(t, u^\varepsilon(t)),$$

with control determined by stochastic optimization procedure (5) and $\{C(y,x,u), G(y,x,u)\} \subset C^{2,0,2}(\mathbb{R}^d, X, \mathbb{R}^d)$.

Then weak convergence

$$(y^\varepsilon(t), u^\varepsilon(t)) \Rightarrow (\hat{y}(t),\hat{u}(t)),\ \varepsilon \to 0,$$

holds true, where ε is small enough, and limit process $(\hat{y}(t),\hat{u}(t))$ is determined by generator

$$\mathbf{A}(y,u)\varphi(y,u) = a(y,u)\varphi'_y(y,u) + \alpha(t)\nabla_{\beta(t)}G(y,u)\varphi'_u(y,u)$$

$$a(y,u) = \int_X \pi(dx)a(y,u,x)$$

on test functions $\varphi(y,u) \in C^{3,2}(\mathbb{R}^d,\mathbb{R}^d)$.

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Article: received 18.09.2020
revised 03.11.2020
printing adoption 24.11.2020

ЗАДАЧА КЕРУВАННЯ ДЛЯ ІМПУЛЬСНОГО ПРОЦЕСУ В УМОВАХ ПРОЦЕДУРИ СТОХАСТИЧНОЇ ОПТИМІЗАЦІЇ ТА НЕКЛАСИЧНИХ СХЕМ АПРОКСИМАЦІЇ

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Для системи еволюційних рівнянь з імпульсним збуренням у схемі некласичної апроксимації побудовано процедуру стохастичної оптимізації та граничний генератор з керуванням, що визначається умовою екстремуму функції критерію якості. Задача керування з використанням процедури стохастичної оптимізації є узагальненням задачі керування за допомогою процедури стохастичної апроксимації, яку досліджували в попередніх працях авторів. Це узагальнення не є простим і потребує нетривіального підходу до вирішення проблеми. Зокрема, ми обговорюємо, як поведінка граничного процесу залежить від дограничної стохастичної еволюційної системи в ергодичному марковському середовищі. Основне припущення – умова рівномірної ергодичності процесу марковського перемикання, тобто існування стаціонарного розподілу для процесу перемикання на великих проміжках часу. Це дає змогу побудувати явні алгоритми для аналізу асимптотичної поведінки керованого процесу. Вперше розроблено модель задачі керування з імпульсним процесом з використанням процедури стохастичної оптимізації задачі керування. У підсумку отримано генератор трикомпонентного марковського процесу, розв'язано задачу сингулярного збурення з поданням граничного генератора цього процесу.

Ключові слова: випадкова еволюція, некласичні схеми апроксимації, процедура стохастичної оптимізації, марковський процес.